Supplementary Online Appendix for
"Overlobbying and Pareto-improving Agenda Constraint" by
A. Dellis and M. Oak (2016)

Expressions for $W_i(\rho)$ and $Z_i(\rho)$

The expressions for $W_i(\rho)$ and $Z_i(\rho)$ depend on whether or not there is an agenda constraint.

When there is no agenda constraint ($N = 2$) the policy choice for issue i depends on only the PM's belief for this issue, $\beta_i(\ell_i, a_i)$. If the PM grants access to IG$_i$, he anticipates to make the correct policy choice for issue i with probability one; $W_i(\rho) = 1$. If the PM grants access to IG$_{-i}$, and therefore not to IG$_i$, he anticipates to make the correct policy choice for issue i with probability

$$Z_i(\rho) = \beta_i^{Acc}\rho_i(1,0) + \left(1 - \beta_i^{Acc}\right)[1 - \rho_i(1,0)]$$

where $\rho_i(1,0)$ denotes the probability that the PM chooses $p_i = 1$ given $(\ell_i, a_i) = (1,0)$. The first term on the r.h.s is the PM's belief that $\theta_i = p_i = 1$. The second term on the r.h.s. is the PM's belief that $\theta_i = p_i = 0$.

We thus obtain the following expression for $X_i(\rho)$:

$$X_i(\rho) = \rho_i(1,0)\ \left(1 - \beta_i^{Acc}\right) + [1 - \rho_i(1,0)]\ \beta_i^{Acc}$$

which corresponds to the PM's belief that he will make the wrong policy choice for issue i if he does not grant access to IG$_i$.

When there is an agenda constraint ($N = 1$) the policy choice for issue i depends on the PM's beliefs for both issues, $\beta_1(\ell_1, a_1)$ and $\beta_2(\ell_2, a_2)$. It is not difficult, although tedious, to show that the expressions for $W_i(\rho)$ and $Z_i(\rho)$ are given in this case by

$$\begin{cases} W_i(\rho) = \beta_i^{Acc}\rho_i(\theta_i = 1) + \left(1 - \beta_i^{Acc}\right) \\ Z_i(\rho) = \left(1 - \beta_i^{Acc}\right) \\ \qquad\qquad + \left(2\beta_i^{Acc} - 1\right)\left[\beta_{-i}^{Acc}\rho_i(\theta_{-i} = 1) + \left(1 - \beta_{-i}^{Acc}\right)\rho_i(\theta_{-i} = 0)\right] \end{cases}$$

where $\rho_i(\theta_i = 1)$ (resp. $\rho_i(\theta_{-i})$) is a shorthand for the probability that the PM chooses $p_i = 1$ given that he had granted access to IG$_i$ (resp. IG$_{-i}$) and observed $\theta_i = 1$ (resp. θ_{-i}) while, at the same time, IG$_{-i}$ (resp. IG$_i$) lobbied but was not granted access.

We thus obtain the following expression for $X_i(\rho)$:

$$X_i(\rho) = \beta_i^{Acc}\rho_i(\theta_i = 1) - \left(2\beta_i^{Acc} - 1\right)\left[\beta_{-i}^{Acc}\rho_i(\theta_{-i} = 1) + \left(1 - \beta_{-i}^{Acc}\right)\rho_i(\theta_{-i} = 0)\right]$$

which corresponds to

(1) the PM's belief that $p_i = \theta_i = 1$ when he grants access to IG$_i$ (first term on the r.h.s) from which we subtract the PM's belief that $p_i = \theta_i = 1$ when he does not grant access to IG$_i$ (β_i^{Acc} times the term in square brackets), and to which we add

(2) the PM's belief that $p_i = 1$ and $\theta_i = 0$ when he does not grant access to IG$_i$ ($\left(1 - \beta_i^{Acc}\right)$ times the term in square brackets).

The first part corresponds to the probability increase of making the correct policy choice for issue i when $\theta_i = 1$. The second part corresponds to the probability reduction of making the wrong policy choice for issue i when $\theta_i = 0$ (knowing that the PM will choose $p_i = 0$ when he grants access to IG$_i$ and observes $\theta_i = 0$).

Extensions

Proof of Lemma 3.

Note that if we had $\delta^* = 0$ then it must be that $\lambda_i(1) = \lambda_i(0) = 0$. In that case an IG$_i$ has a profitable deviation $\lambda_i(0) = 1$. This gives us a contradiction.

Similarly, if we had $\delta^* = 1$ then it must be that $\lambda_i(1) = \lambda_i(0) = 1$. However, if we had an equilibrium with $\lambda^*(0) = \lambda^*(1) = 1$ then it must be that $\beta^*(1, \phi) = \pi(< 1/2)$, which means we must have $\rho^*(1, \phi) = 0$. However, in that case i is better off deviating to $\lambda_i(0) = 0$ and theyby saving on the lobbying cost. This gives us a contradiction. Hence, in any symmetric equilibrium we must have $\delta^* \in (0, 1)$.

Since $\delta^* > 0$, we have

$$\frac{\partial E\Pi_i}{\partial \lambda_i(1)} = (1 - \Gamma_i(\delta^*)) \cdot \rho_i^*(1, \phi) + \Gamma_i(\delta^*) - f > \frac{\partial E\Pi_i}{\partial \lambda_i(0)} = (1 - \Gamma_i(\delta^*)) \cdot \rho_i^*(1, \phi) - f.$$

Since $\delta^* > 0 \Longrightarrow \Gamma^*(\delta^*) > 0$, we have that $\lambda^*(1) \geq \lambda^*(0)$. But we already ruled out the case, $\lambda^*(1) = \lambda^*(0) = 1$. Hence, we must have $\lambda^*(1) > \lambda^*(0)$.

This gives us three possible types of symmetric equilibria: (1) truthful lobbying equilibrium: $\lambda^*(1) = 1, \lambda^*(0) = 0$; (2) overlobbying equilibrium: $\lambda^*(1) = 1, \lambda^*(0) \in (0, 1)$; (3) underlobbying equilibrium: $\lambda^*(1) \in (0, 1), \lambda^*(0) = 0$.■

Proof of Lemma 4.

4.1 *if* part. Suppose $f \in [(1 - \Gamma(\pi)) \cdot \widetilde{\rho}(\pi), (1 - \Gamma(\pi)) \cdot \widetilde{\rho}(\pi) + \Gamma(\pi)]$. Consider the truthful lobbying strategies played by the IGs (i.e., $\lambda(0) = 0, \lambda(1) = 1$); let access strategy of the PM be $\gamma(0) = 0, \gamma(1) = \min\{K/M, 1\}$ where $M = \#i$ such that $\ell_i = 1$; and let $\rho(0, \cdot) = 0 = \rho(1, 0), \rho(1, 1) = 1$ and $\rho(1, \phi) = \max\{\frac{N - \widehat{K}}{M - K}, 1\}$ where $M = \#i$ such that $\ell_i = 1$ and $\widehat{K} = \#i$ such that $m_i = 1$; PM's beliefs are $\beta^{Acc}(0) = 0, \beta^{Acc}(1) = 1; \beta(0) = 0, \beta(1) = 1$ and $\beta(\phi) = 1$. It is easy to see that the interim beliefs are consistent with the lobbying strategies; the access strategy is optimal given the beliefs and the policy function; the final beliefs are consistent with the access strategy and the policy function; and the policy function if optimal given the beliefs. We, therefore need to check if each IG$_i$'s lobbying strategies are optimal given the PM's and other IG's strategies. Note that when $\theta_i = 0$, IG$_i$'s expected payoff from lobbying is $(1 - \Gamma(\pi)) \cdot \widetilde{\rho}(N, \pi) - f$ which is weakly less 0, the expected payoff from not lobbying. On the other hand, when $\theta_i = 1$, IG$_i$'s expected payoff from lobbying is $(1 - \Gamma(\pi)) \cdot \widetilde{\rho}(N, \pi) + \Gamma(\pi) - f$ which is weakly greater than 0, the expected payoff from not lobbying. This establishes that the strategies described above constitute a symmetric Nash equilibrium.

4.1 *only if* part. Given the definition of equilibrium, a truthful lobbying equilibrium is unique. In equilibrium an IG$_i$ lobbies with probability 1 when $\theta_i = 1$ which implies $f \leq (1 - \Gamma(\pi)) \cdot \widetilde{\rho}(N, \pi) + \Gamma(\pi)$; also, the IG$_i$ does not lobby when $\theta_i = 0$, which implies $f \geq (1 - \Gamma(\pi)) \cdot \widetilde{\rho}(N, \pi)$.

4.2 *Overlobbying equilibrium:* There are two possibilities to consider: 4.2.1 There exists $\widehat{\delta} \in [\pi, 2\pi]$ such that $f = [1 - \Gamma(\widehat{\delta})] \cdot \widetilde{\rho}(N, \widehat{\delta})$. 4.2.2 For any $\delta \in [\pi, 2\pi]$, $f > [1 - \Gamma(\delta)] \cdot \widetilde{\rho}(N, \delta)$.

4.2.1 Pick such a $\widehat{\delta}$. Consider the following strategies for each IG: $\lambda(1) = 1$, $\lambda(0) = \widehat{\delta} - \pi/1 - \pi$; PM's strategies are $\gamma(0) = 0, \gamma(1) = \min\{K/M, 1\}$ where $M = \#i$ such that $\ell_i = 1$; and $\rho(0, \cdot) = 0 = \rho(1, 0), \rho(1, 1) = 1$ and $\rho(1, \phi) = \max\{\frac{N-\widehat{K}}{M-K}, 1\}$ where $M = \#i$ such that $\ell_i = 1$ and $\widehat{K} = \#i$ such that $m_i = 1$. PM's interim and final beliefs are $\beta^{Acc}(0) = 0$, $\beta^{Acc}(1) = \pi/\widehat{\delta}$; $\beta(0) = 0$, $\beta(1) = 1$ and $\beta(\phi) = \pi/\widehat{\delta}$. Since $\pi/\widehat{\delta} \geq 1/2$, PM's policy choice is optimal given his beliefs. Also, his access strategy is optimal given the IGs' lobbying strategies and his interim beliefs; moreover, PM's interim belifes are derived from the IGs' strategies using Bayes Rule. Also, $f = [1-\Gamma(\widehat{\delta})] \cdot \widetilde{\rho}(N, \widehat{\delta}) \Longrightarrow \lambda(0) = \widehat{\delta} - \pi/1 - \pi$ is optimal and also, $f < [1 - \Gamma(\widehat{\delta})] \cdot \widetilde{\rho}(N, \widehat{\delta}) + \Gamma(\widehat{\delta}) \Longrightarrow \lambda(1) = 1$ is optimal. This establishes that the strategies, and beliefs described above constitute a symmetric Nash equilibrium.

4.2.2 Consider the following strategies for the IGs: $\lambda(1) = 1, \lambda(0) = \pi/1 - \pi$; PM's strategies are $\gamma(0) = 0, \gamma(1) = \min\{K/M, 1\}$ where $M = \#i$ such that $\ell_i = 1$; and $\rho(0, \cdot) = 0 = \rho(1, 0), \rho(1, 1) = 1$ and $\rho(1, \phi) = \alpha \cdot \max\{\frac{N-\widehat{K}}{M-K}, 1\}$ where $M = \#i$ such that $\ell_i = 1$ and $\widehat{K} = \#i$ such that $m_i = 1$, where α is chosen such that $f = [1 - \Gamma(2\pi)] \cdot \alpha \cdot \widetilde{\rho}(N, 2\pi)$. PM's interim and final beliefs are $\beta^{Acc}(0) = 0$, $\beta^{Acc}(1) = 1/2$; $\beta(0) = 0$, $\beta(1) = 1$ and $\beta(\phi) = 1/2$. Note that given the PM's belief $\beta(\phi) = 1/2$, he is indifferent between implementing and not implementing reform on an issue to which he did not grant access inspite of lobbying. Hence, his strategy to implement reforms on a fraction α of such issues is optimal. Likewise, an IG is indifferent between lobbying an not lobbying in state 0 since $f = [1 - \Gamma(2\pi)] \cdot \alpha \cdot \widetilde{\rho}(N, 2\pi)$. This establishes that the strategies and beliefs described above constitute a symmetric Nash equilibrium.

4.2 *Underlobbying equilibrium:* Suppose $f > [1 - \Gamma(\pi)] \cdot \widetilde{\rho}(N, \pi) + \Gamma(\pi)$. Observe that $[1 - \Gamma(\delta)] \cdot \widetilde{\rho}(N, \delta) + \Gamma(\delta)$ is a continuous function of δ and as $\delta \to 0$, $[1 - \Gamma(\delta)] \cdot \widetilde{\rho}(N, \delta) + \Gamma(\delta) \to 1 < f$. Hence, there exists $\widehat{\delta} \in (0, \pi)$ such that $[1 - \Gamma(\widehat{\delta})] \cdot \widetilde{\rho}(N, \widehat{\delta}) + \Gamma(\widehat{\delta}) = f$. Consider the following strategies for the IGs: $\lambda(1) = \widehat{\delta}/\pi, \lambda(0) = 0$; PM's strategies are $\gamma(0) = 0, \gamma(1) = \min\{K/M, 1\}$ where $M = \#i$ such that $\ell_i = 1$; and $\rho(0, \cdot) = 0 = \rho(1, 0), \rho(1, 1) = 1$ and $\rho(1, \phi) = \max\{\frac{N-\widehat{K}}{M-K}, 1\}$ where $M = \#i$ such that $\ell_i = 1$ and $\widehat{K} = \#i$ such that $m_i = 1$. PM's interim and final beliefs are $\beta^{Acc}(0) = \frac{\pi(1-\widehat{\delta})}{\pi(1-\widehat{\delta})+(1-\pi)}$, $\beta^{Acc}(1) = 1$; $\beta(0) = 0$, $\beta(1) = 1$ and $\beta(\phi) = 1$.As in the case of truthful lobbying equilibrium, PM's policy choice is optimal given his beliefs. Also, his access strategy is optimal given the IGs' lobbying strategies and his interim beliefs; moreover, PM's interim belifes are derived from the IGs' strategies using Bayes Rule. This establishes that the strategies and beliefs described above constitute a symmetric Nash equilibrium.

4.3 Suppose there existed a symmetric overlobbying equilibrium for $f \geq \overline{f}(N)$. Let $\delta^* \in (\pi, 2\pi]$ denote the ex-ante lobbying probability in such equilibrium. It must then be the case that $[1 - \Gamma(\delta^*)] \cdot \alpha \cdot \widetilde{\rho}(N, \delta^*) + \Gamma(\delta^*) > f$ for some $\alpha \in (0, 1]$, which in turn implies that $[1-\Gamma(\delta^*)] \cdot \widetilde{\rho}(N, \delta^*) + \Gamma(\delta^*) > f$. However, since $[1-\Gamma(\delta)] \cdot \widetilde{\rho}(N, \delta) + \Gamma(\delta)$ is decreasing on $(0, 2\pi]$, we have $\overline{f}(N) \equiv [1 - \Gamma(\pi)] \cdot \widetilde{\rho}(N, \pi) + \Gamma(\pi) > [1 - \Gamma(\delta^*)] \cdot \widetilde{\rho}(N, \delta^*) + \Gamma(\delta^*) > f$. This gives us a contradiction.

Similarly, suppose there existed a symmetric underlobbying equilibrium for $f \leq \overline{f}(N)$. Let $\delta^* \in (0, \pi)$ denote the ex-ante lobbying probability in such equilibrium. Such equilibrium requires that $[1 - \Gamma(\delta^*)] \cdot \widetilde{\rho}(N, \delta^*) + \Gamma(\delta^*) = f$. But given that

$[1-\Gamma(\delta)]\cdot\widetilde{\rho}(N,\delta)+\Gamma(\delta)$ is decreasing, we have $[1-\Gamma(\pi)]\cdot\widetilde{\rho}(N,\pi)+\Gamma(\pi)\equiv\overline{f}(N)<f$. This gives us a contradiction.■

Proof of Proposition 3.

To prove this proposition we establish a series of claims.

Claim 1: For any $N\in\{K,\cdots,I\}$, $\overline{f}(N)-\underline{f}(N)$ is positive and constant independent of N.

To see this note that $\overline{f}(N)-\underline{f}(N)$ can be broken down as

$$\begin{aligned}&\sum_{n=0}^{K-1}z_n(\pi)+\sum_{n=K}^{N-1}[1-\frac{n+1-K}{n+1}]\cdot z_n(\pi)+\sum_{n=N}^{I-1}[\frac{N}{n+1}-\frac{N-K}{n+1}]\cdot z_n(\pi)\\ =\ &\sum_{n=0}^{K-1}z_n(\pi)+\sum_{n=K}^{I-1}\frac{K}{n+1}\cdot z_n(\pi)>0\end{aligned}$$

since each term is non-negative and at least one term is strictly positive. In fact the expression above is $\Gamma^*(\pi)$ which is independent of N.

Claim 2: $\underline{f}(N)$ is an increasing function of N.

To see this note that $\underline{f}(N)-\underline{f}(N+1)$ can be broken down as

$$\begin{aligned}&-\frac{N+1-K}{N+1}\cdot z_N(\pi)+\frac{N-K}{N+1}\cdot z_N(\pi)+\sum_{n=N+1}^{I-1}\frac{-(N+1-K)+(N=K)}{n+1}\cdot z_n(\pi)\\ =\ &-\sum_{n=N}^{I-1}\frac{1}{n+1}\cdot z_n(\pi)<0.\end{aligned}$$

We can also see that when $N=I$ we have $\overline{f}(N)=1$ and $\underline{f}(N)=1-\Gamma^*(\pi)$. Similarly, when $N=K$ we have $\overline{f}(N)=\Gamma^*(\pi)$ and $\underline{f}(N)=0$. This establishes Proposition 3.■

Proof of Lemma 5. We start by stating IG$_i$'s lobbying problem. Given θ_i, IG$_i$ chooses lobbying strategy $\lambda_i(\theta_i)$ that solves

$$\max_{\lambda_i(\theta_i)\in[0,1]}Ev_i(\lambda_i(\theta_i))$$

where

$$\begin{aligned}Ev_i(\lambda_i(\theta_i)) &= \lambda_i(\theta_i)[\Gamma_i(1)\rho_i(1,1)+(1-\Gamma_i(1))\rho_i(1,0)-f_i]\\ &\quad+(1-\lambda_i(\theta_i))[\Gamma_i(0)\rho_i(0,1)+(1-\Gamma_i(0))\rho_i(0,0)]\end{aligned}$$

and the probability that IG$_i$ is granted access given lobbying decision $\ell_i\in\{0,1\}$ is

$$\Gamma_i(\ell_i)\equiv\delta_{-i}\gamma_i(\ell_i,1)+(1-\delta_{-i})\gamma_i(\ell_i,0).$$

We prove part (1) of the statement. We first establish the sufficiency of the condition. Suppose that the condition is satisfied. Let $\lambda_i(1)=1$ and $\lambda_i(0)=0$ for each $i=1,2$. It follows that $\delta_i=\pi_i$ for each $i=1,2$, and that the PM is perfectly informed about θ through the lobbying decisions. Specifically, $\beta_i^{Acc}(1)=1$ and $\beta_i^{Acc}(0)=0$ for each $i=1,2$. Denoting by $X_i(\ell_i;\rho)$ the increase in the probability that the PM will make the correct policy choice on issue i by granting access to IG$_i$ when its lobbying decision is ℓ_i, we get $X_i(\ell_i;\rho)=0$ for each $i=1,2$ and each $\ell_i=0,1$. The PM is thus indifferent granting access to IG_1 or IG_2.

Using the PM's policy choice strategy described in section 3, we get

$$\begin{cases} \frac{dEv_i}{d\lambda_i(1)} \geq 0 \Leftrightarrow 1 - \Gamma_i(0) - f_i \geq 0 \\ \frac{dEv_i}{d\lambda_i(0)} \leq 0 \Leftrightarrow 1 - \Gamma_i(1) - f_i \leq 0. \end{cases}$$

In order to minimize $\Gamma_i(0)$ and maximize $\Gamma_i(1)$, we set $\gamma_1(1,0) = \gamma_2(0,1) = 1$. This is possible since $X_i(\ell_i; \rho) = 0$. The two conditions above are then satisfied if and only if

$$\begin{cases} \gamma_1(1,1) = [1 - \gamma_2(1,1)] \in \left[1 - \frac{f_1}{\pi_2}, \frac{f_2}{\pi_1}\right] \\ \gamma_1(0,0) = [1 - \gamma_2(0,0)] \in \left[\frac{f_2 - \pi_1}{1 - \pi_1}, \frac{1 - f_1}{1 - \pi_2}\right]. \end{cases}$$

Given the condition in the statement of the lemma, these two intervals are non-empty. Moreover, the lower-bounds (resp. upper-bounds) of the two intervals are smaller than one (resp. bigger than zero). Hence, there exists a strategy profile $\{\lambda(.), \gamma(.), \rho(.)\}$ that constitutes an equilibrium of the lobbying subgame in which the PM gets perfectly informed about θ.

We continue by establishing the necessity of the condition in the statement. Suppose that this condition is not satisfied. Assume by way of contradiction that an equilibrium exists in which the PM gets perfectly informed about θ. Since the PM can grant access to only one IG, the lobbying decisions of at least one IG must be perfectly informative. Formally, $|\lambda_i(1) - \lambda_i(0)| = 1$ for some $i \in \{1,2\}$. We must then have $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$. Moreover, $\lambda_i(0) = 0$ requires

$$\frac{dEv_i}{d\lambda_i(0)} \leq 0 \Leftrightarrow \Gamma_i(1) \geq 1 - f_i.$$

Hence it must be that $\Gamma_i(1) > 0$ and, therefore, that $\gamma_i(1, \ell_{-i}) > 0$ for some $\ell_{-i} \in \{0,1\}$. Given that truthful lobbying by IG$_i$ implies $X_i(1;\rho) = 0$, $\Gamma_i(1) > 0$ requires $\lambda_{-i}(1) \neq \lambda_{-i}(0)$ and $\beta_{-i}^{Acc}(\ell_{-i}) \in \{0,1\}$ for some ℓ_{-i}. There are three cases to consider:

(1) $\lambda_{-i}(1) = 1$ and $\lambda_{-i}(0) \in (0,1)$. In this case, $\beta_{-i}^{Acc}(0) = 0$ and $\beta_{-i}^{Acc}(1) \in (0,1)$, implying $\rho_{-i}(0,0) = 0$ and $\Gamma_{-i}(1) = 1$. We then get $\frac{dEv_{-i}}{d\lambda_{-i}(0)} = -f_{-i} < 0$, contradicting $\lambda_{-i}(0) > 0$.

(2) $\lambda_{-i}(1) \in (0,1)$ and $\lambda_{-i}(0) = 0$. In this case, $\beta_{-i}^{Acc}(1) = 1$ and $\beta_{-i}^{Acc}(0) \in (0,1)$, implying $\rho_{-i}(1,0) = 1$ and $\Gamma_{-i}(0) = 1$. We then get $\frac{dEv_{-i}}{d\lambda_{-i}(1)} = -f_{-i} < 0$, contradicting $\lambda_{-i}(1) > 0$.

(3) $\lambda_{-i}(1) = 1$ and $\lambda_{-i}(0) = 0$. In this case, we have $\lambda_i(1) = 1$ and $\lambda_i(0) = 0$ for each i, which yields a contradiction given that the condition in the statement is not satisfied. We can establish the contradiction by proceeding in a way similar to the way we proceeded for establishing the sufficiency of the condition.

This completes the proof since these three cases exhaust all possibilities.

We now prove part (2) of the statement. We start by identifying a series of conditions that are necessary for the existence of an equilibrium with $\delta_i > 0$ for $i = 1,2$.

First, we establish that $\beta_i^{Acc}(0) < 1/2$ and, therefore, that $\rho_i(0,0) = 0$ for each $i = 1,2$. Assume by way of contradiction that $\beta_i^{Acc}(0) \geq 1/2$ for some i. Given that $\pi_i < 1/2$, it must be that either $\lambda_i(1) = \lambda_i(0) = 1$ or $\lambda_i(0) > \lambda_i(1)$. In either case, $\lambda_i(0) > 0$ and $\beta_i^{Acc}(1) < 1/2$. The latter implies $\rho_i(1,0) = 0$. It follows that $\frac{dEv_i}{d\lambda_i(0)} \leq -f_i < 0$, which contradicts $\lambda_i(0) > 0$.

Second, we establish that $\beta_i^{Acc}(1) \geq 1/2$ for each $i = 1, 2$. Assume by way of contradiction that $\beta_i^{Acc}(1) < 1/2$ for some i. It follows that $\rho_i(1,0) = 0$ which, together with $\rho_i(0,0) = 0$, implies $\frac{dEv_i}{d\lambda_i(0)} = -f_i < 0$ and, therefore, $\lambda_i(0) = 0$. Since $\delta_i > 0$, we then get $\lambda_i(1) > 0$ and $\beta_i^{Acc}(1) = 1$, which contradicts $\beta_i^{Acc}(1) < 1/2$.

Third, we establish that $\beta_1^{Acc}(1) > 1/2$ and, therefore, that $\rho_1(1,0) = 1$. We already know from above that $\beta_1^{Acc}(1) \geq 1/2$. Assume by way of contradiction that $\beta_1^{Acc}(1) = 1/2$. It follows that $X_1(1;\rho) = 1/2$. Given that $\alpha > 1$ and $X_2(\ell_2;\rho) = \left[1 - \beta_2^{Acc}(1)\right] \leq 1/2$ for each $\ell_2 = 0, 1$, we get $X_1(1;\rho)\,\alpha > X_2(\ell_2;\rho)$ for each $\ell_2 = 0, 1$, implying $\Gamma_1(1) = 1$. It follows that $\frac{dEv_1}{d\lambda_1(0)} = -f_1 < 0$ and, therefore, $\lambda_1(0) = 0$. Since $\delta_1 > 0$, we get $\lambda_1(1) > 0$ and $\beta_1^{Acc}(1) = 1$, contradicting $\beta_1^{Acc}(1) = 1/2$.

Fourth, we establish that $\lambda_i(0) > 0$ for some $i \in \{1, 2\}$. Assume by way of contradiction that $\lambda_i(0) = 0$ for each $i = 1, 2$. Since $\delta_i > 0$, we then get $\lambda_i(1) > 0$ and, therefore, $\beta_i^{Acc}(1) = 1$ and $\rho_i(1,0) = 1$.

We start by showing that we must then have $\lambda_i(1) \in (0,1)$ for each $i = 1, 2$. Assume by way of contradiction that $\lambda_h(1) = 1$ for some $h \in \{1, 2\}$. Since $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$, we know from part (1) of this lemma that we cannot have truthful lobbying strategies. We must then have $\lambda_{-h}(1) \in (0,1)$. It follows that $X_{-h}(0;\rho) > 0 = X_h(0;\rho) = X_h(1;\rho)$ and, therefore, $\Gamma_{-h}(0) = 1$. We then have $\frac{dEv_{-h}}{d\lambda_{-h}(1)} = -f_h < 0$ and, therefore, $\lambda_{-h}(1) = 0$, contradicting $\lambda_{-h}(1) \in (0,1)$.

We continue by observing that $\lambda_i(1) \in (0,1)$ and $\lambda_i(0) = 0$ for each $i = 1, 2$ has two implications. First, we have $X_i(0;\rho) > 0 = X_i(1;\rho)$ for each $i = 1, 2$, which implies $\gamma_1(0,1) = \gamma_2(1,0) = 1$. It follows that $\Gamma_i(0) \geq \Gamma_i(1)$ for each $i = 1, 2$. Second, it must be that

$$\begin{cases} \frac{dEv_i}{d\lambda_i(1)} = 0 \Leftrightarrow \Gamma_i(0) = 1 - f_i \\ \frac{dEv_i}{d\lambda_i(0)} \leq 0 \Leftrightarrow \Gamma_i(1) \geq 1 - f_i, \end{cases}$$

implying $\Gamma_i(1) \geq \Gamma_i(0)$ for each $i = 1, 2$. We get from these two implications that $\Gamma_i(1) = \Gamma_i(0)$ for each $i = 1, 2$. Since $\gamma_1(0,1) = \gamma_2(1,0) = 1$, it must then be that $\gamma_i(1,1) = 1$ and $\gamma_i(0,0) = 0$ for each $i = 1, 2$, a contradiction.

Fifth, we establish that $\lambda_i(0) > 0$ for each $i = 1, 2$. Assume by way of contradiction that $\lambda_i(0) = 0$ for some $i \in \{1, 2\}$. This has two sets of implications. One set of implications is that $\lambda_i(1) > 0$, $\beta_i^{Acc}(1) = 1$, $\rho_i(1,0) = 1$ and $X_i(1;\rho) = 0$. The other set of implications is that $\lambda_{-i}(0) > 0$ (from above). Since $\beta_{-i}^{Acc}(1) \geq 1/2$ and $\pi_{-i} < 1/2$, it must be that $\lambda_{-i}(1) > \lambda_{-i}(0)$ and $X_{-i}(1;\rho) > 0$. It follows from these two sets of implications that $X_{-i}(1;\rho) > 0 = X_i(1;\rho)$ and, therefore, that $\gamma_{-i}(1,1) = 1$.

We continue by showing that we must have $\lambda_{-i}(1) = 1$. Given that $\lambda_i(0) = 0$, we have

$$\frac{dEv_i}{d\lambda_i(0)} \leq 0 \Leftrightarrow \Gamma_i(1) \geq 1 - f_i,$$

implying $\Gamma_i(1) > 0$. Given that $\gamma_{-i}(1,1) = 1$, it must be that $\gamma_i(1,0) > 0$ (where $\gamma_i(\ell_i, \ell_{-i})$) and, therefore, that $X_i(1;\rho)\alpha_i \geq X_{-i}(0;\rho)\alpha_{-i}$. Since $X_i(1;\rho) = 0$, it must then be that $X_{-i}(0;\rho) = 0$ and, therefore, that $\lambda_{-i}(1) = 1$.

We further show that we must have $\lambda_i(1) \in (0,1)$. Given that $\lambda_{-i}(0) \in (0,1)$, we have

$$\frac{dEv_{-i}}{d\lambda_{-i}(0)} = 0 \Leftrightarrow [1 - \Gamma_{-i}(1)]\,\rho_{-i}(1,0) = f_{-i},$$

implying $\Gamma_{-i}(1) < 1$. Given that $\gamma_{-i}(1,1) = 1$, it must then be that $\gamma_{-i}(0,1) < 1$ and, therefore, that $X_i(0;\rho)\alpha_i \geq X_{-i}(1;\rho)\alpha_{-i}$. Since $X_{-i}(1;\rho) > 0$, it must then be that $X_i(0;\rho) > 0$ and, therefore, that $\lambda_i(1) < 1$.

We continue by observing that $X_i(0;\rho) > 0 = X_i(1;\rho)$ and $X_{-i}(0;\rho) = 0 < X_{-i}(1;\rho)$ imply $\gamma_i(0,0) = \gamma_{-i}(1,1) = 1$. Moreover, $\lambda_i(1) \in (0,1)$ and $\lambda_i(0) = 0$ require

$$\begin{cases} \frac{dEv_i}{d\lambda_i(1)} = 0 \Leftrightarrow \Gamma_i(0) = 1 - f_i \\ \frac{dEv_i}{d\lambda_i(0)} \leq 0 \Leftrightarrow \Gamma_i(1) \geq 1 - f_i, \end{cases}$$

implying $\Gamma_i(1) \geq \Gamma_i(0)$. Since $\gamma_i(0,0) = 1$ and $\gamma_i(1,1) = 0$, it must then be that $\gamma_i(1,0) = 1$ and $\gamma_i(0,1) = 0$. But then, we have $\Gamma_{-i}(1) = 1$ and, therefore, $\frac{dEv_{-i}}{d\lambda_{-i}(0)} = -f_{-i} < 0$, contradicting $\lambda_{-i}(0) > 0$.

Sixth, we establish that $\lambda_2(1) = 1$. Assume by way of contradiction that $\lambda_2(1) < 1$. It follows that $1 > \lambda_2(1) > \lambda_2(0) > 0$, which requires

$$\begin{cases} \frac{dEv_2}{d\lambda_2(1)} = 0 \Leftrightarrow \Gamma_2(1) - \Gamma_2(0) + (1 - \Gamma_2(1))\,\rho_2(1,0) = f_2 \\ \frac{dEv_2}{d\lambda_2(0)} = 0 \Leftrightarrow (1 - \Gamma_2(1))\,\rho_2(1,0) = f_2, \end{cases}$$

implying $\Gamma_2(0) = \Gamma_2(1) < 1$. Moreover, we have $X_2(\ell_2;\rho) > 0$ for each $\ell_2 = 0, 1$.

Also, recall from above that $\rho_1(1,0) = 1$ and $\lambda_1(0) \in (0,1)$, the latter requiring

$$\frac{dEv_1}{d\lambda_1(0)} = 0 \Leftrightarrow \Gamma_1(1) = 1 - f_1.$$

We shall establish the contradiction in two steps, first for $\lambda_1(1) = 1$ and then for $\lambda_1(1) < 1$.

Suppose $\lambda_1(1) = 1$. Since $\lambda_1(0) \in (0,1)$, we then have $\beta_1^{Acc}(0) = 0$, which implies $\rho_1(0,0) = 0$ and $X_1(0;\rho) = 0$. Given that $X_2(\ell_2;\rho) > 0$ for each $\ell_2 = 0, 1$, we then have $X_2(\ell_2;\rho) > X_1(0;\rho)\alpha$ and, therefore, $\gamma_2(0,0) = \gamma_2(0,1) = 1$. The latter implies $\gamma_2(1,0) = \gamma_2(1,1) = f_1 \in (0,1)$, the first equality since $\Gamma_2(0) = \Gamma_2(1)$ and the second equality since $\Gamma_1(1) = 1 - f_1$. For $\gamma_2(1,\ell_2) \in (0,1)$, it must be that $X_2(\ell_2;\rho) = X_1(1;\rho)\alpha$. Now, observe that $X_2(0;\rho) = \beta_2^{Acc}(0) < 1/2$. It follows that $X_2(1;\rho) < 1/2$, which is true only if $\beta_2^{Acc}(1) > 1/2$ and, therefore, $\rho_2(1,0) = 1$. We then get from $\frac{dEv_2}{d\lambda_2(0)} = 0$ that $\Gamma_2(1) = 1 - f_2$ and, therefore, that

$$f_2 = \delta_1(1 - f_1) \Leftrightarrow \lambda_1(0) = \frac{1}{1 - \pi_1}\left[\frac{f_2}{1 - f_1} - \pi_1\right].$$

Hence the contradiction since $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$ implies $\lambda_1(0) < 0$.

Now, suppose $\lambda_1(1) < 1$. It follows that $1 > \lambda_1(1) > \lambda_1(0) > 0$, which requires

$$\frac{dEv_1}{d\lambda_1(1)} = 0 \Leftrightarrow \Gamma_1(0) = 1 - f_1.$$

Recall from above that $\lambda_1(0) \in (0,1)$ implies $\Gamma_1(1) = 1 - f_1$. It follows that $\Gamma_1(0) = \Gamma_1(1) = (1 - f_1) \in (0,1)$.

We consider two cases:

(1) $\beta_2^{Acc}(1) > 1/2$. We then have $\rho_2(1,0) = 1$ and, therefore, $\Gamma_2(0) = \Gamma_2(1) = (1 - f_2)$. It follows that IG_1 is granted access with probability $(1 - f_1)$ and IG_2 with probability $(1 - f_2)$. Since these two probabilities must sum to 1, we have $f_1 + f_2 = 1$, which contradicts $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$.

(2) $\beta_2^{Acc}(1) = 1/2$. We then have $X_2(1;\rho) = 1/2 > \beta_2^{Acc}(0) = X_2(0;\rho)$. For $\Gamma_1(\ell_1) \in (0,1)$ for each $\ell_1 = 0,1$, it must then be that $X_2(1;\rho) \geq X_1(\ell_1;\rho)\alpha \geq X_2(0;\rho)$ for each $\ell_1 = 0,1$. We show that the two inequalities are strict.

Assume by way of contradiction that $X_2(1;\rho) = X_1(\ell_1;\rho)\alpha > X_2(0;\rho)$ for some $\ell_1 \in \{0,1\}$. It follows that $\gamma_1(\ell_1,0) = 1$ and, since $\Gamma_1(\ell_1) < 1$, that $\gamma_1(\ell_1,1) < 1$. The latter implies $\Gamma_2(0) = \Gamma_2(1) > 0$. This, together with $\gamma_1(\ell_1,0) = 1$, implies $\gamma_2(\sim\ell_1,0) > 0$ and, therefore, $X_2(0;\rho) \geq X_1(\sim\ell_1;\rho)\alpha$. It follows that $X_2(1;\rho) > X_1(\sim\ell_1;\rho)\alpha$, which implies $\gamma_2(\sim\ell_1,1) = 1$. The latter, together with $\gamma_2(\ell_1,1) > 0$ and $\gamma_2(\ell_1,0) = 0$, implies $\Gamma_2(1) > \Gamma_2(0)$, which contradicts $\Gamma_2(1) = \Gamma_2(0)$.

Assume by way of contradiction that $X_2(1;\rho) > X_1(\ell_1;\rho)\alpha = X_2(0;\rho)$ for some $\ell_1 \in \{0,1\}$. It follows that $\gamma_2(\ell_1,1) = 1$ and, since $\Gamma_2(1) < 1$, that $\gamma_2(\sim\ell_1,1) < 1$. The latter implies $X_1(\sim\ell_1;\rho)\alpha \geq X_2(1;\rho)$. It follows that $X_1(\sim\ell_1;\rho)\alpha > X_2(0;\rho)$, which implies $\gamma_2(\sim\ell_1,0) = 0$. This, together with $\gamma_2(\ell_1,1) = 1$ and $\Gamma_2(1) = \Gamma_2(0)$, implies $\gamma_2(\ell_1,0) = 1$ and $\gamma_2(\ell_1,1) = 0$. We then get $\Gamma_1(\ell_1) = 0$ and $\Gamma_1(\sim\ell_1) = 1$, which contradicts $\Gamma_1(\ell_1) = \Gamma_1(\sim\ell_1)$.

Thus, we have $X_2(1;\rho) > X_1(\ell_1;\rho)\alpha > X_2(0;\rho)$ for each $\ell_1 = 0,1$. It follows that $\Gamma_2(1) = 1$ and $\Gamma_2(0) = 0$, which contradicts $\Gamma_2(0) = \Gamma_2(1)$.

This completes the proof since these two cases exhaust all possibilities.

Seventh, we establish $\beta_2^{Acc}(1) = 1/2$ and $\lambda_2(0) = \frac{\pi_2}{1-\pi_2}$. Assume by way of contradiction that $\beta_2^{Acc}(1) > 1/2$ and, therefore, that $\rho_2(1,\emptyset) = 1$. It follows that $\lambda_2(0) \in (0,1)$ requires $\Gamma_2(1) = 1 - f_2$. At the same time, we already know that $\lambda_1(0) \in (0,1)$ requires $\Gamma_1(1) = 1 - f_1$. Moreover, $1 = \lambda_2(1) > \lambda_2(0) > 0$ implies $X_2(0;\rho) = 0$. At the same time, $\lambda_1(1) > \lambda_1(0) > 0$ implies $X_1(1;\rho) > 0$. It follows that $X_1(1;\rho)\alpha > X_2(0;\rho)$ and, therefore, that $\gamma_1(1,0) = 1$.

All the above imply

$$\Gamma_1(1) = 1 - f_1 \Leftrightarrow \gamma_1(1,1) = 1 - \frac{f_1}{\delta_2}$$

and

$$(*) \qquad \Gamma_1(1) + f_1 = \Gamma_2(1) + f_2 \Leftrightarrow 1 = \frac{\delta_1 f_1}{\delta_2} + (1-\delta_1)\gamma_2(0,1) + f_2.$$

There are two cases to consider:

(1) $\lambda_1(1) = 1$. In this case, $X_1(0;\rho) = 0$ and, therefore, $\gamma_2(0,1) = 1$ (since $X_2(1;\rho) > 0$). Plugging the value of $\gamma_2(0,1)$ into $(*)$ gives $\frac{\delta_1 f_1 + \delta_2 f_2}{\delta_1 \delta_2} = 1$, which contradicts $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$ and $\delta_i > \pi_i$ for each $i = 1,2$.

(2) $\lambda_1(1) < 1$. In this case, $X_1(0;\rho) > 0$ and, therefore, $\gamma_1(0,0) = 1$ (since $X_2(0;\rho) = 0$). Moreover, $\lambda_1(1) \in (0,1)$ requires

$$\frac{dEv_1}{d\lambda_1(1)} = 0 \Leftrightarrow \Gamma_1(0) = 1 - f_1,$$

which implies $\gamma_2(0,1) = \frac{f_1}{\delta_2}$ (since $\gamma_1(0,0) = 1$). Plugging the value of $\gamma_2(0,1)$ into $(*)$ gives $\frac{f_1}{\delta_2} + f_2 = 1$, which contradicts $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$ and $\delta_2 > \pi_2$.

Hence $\beta_2^{Acc}(1) = 1/2$. This, together with $\lambda_2(1) = 1$, implies $\lambda_2(0) = \frac{\pi_2}{1-\pi_2}$.

To sum up, we have shown that in any equilibrium with $\delta_i > 0$ for $i = 1, 2$, we must have

$$\begin{cases} 1 \geq \lambda_1(1) > \lambda_1(0) > 0 \\ \lambda_2(1) = 1 \text{ and } \lambda_2(0) = \frac{\pi_2}{1-\pi_2} \\ \beta_1^{Acc}(1) > \beta_2^{Acc}(1) = \frac{1}{2} > \beta_1^{Acc}(0) \geq \beta_2^{Acc}(0) = 0 \\ X_2(0;\rho) = 0 \text{ and } X_2(1;\rho) = 1/2 \\ \rho_1(1,0) = 1 \text{ and } \rho_1(0,0) = \rho_2(0,0) = 0. \end{cases}$$

Hence, there are two possible types of equilibria with $\delta_i > 0$ for $i = 1, 2$, namely, those where $\lambda_1(1) = 1$ and those where $\lambda_1(1) < 1$.

We start by considering those equilibria where $\lambda_1(1) = 1$. Proceeding as in the proof of lemma 1, we can establish: 1) such equilibria exist given $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$; and 2) the equilibrium strategies and beliefs are the same as in part (2) of lemma 1, except for $\gamma_i(0,0)$ which can take any value in $[0,1]$ and must satisfy the condition that $\sum_{i=1}^{2} \gamma_i(0,0) = 1$. These equilibria differ only in $\gamma_i(0,0)$.

It remains to consider the possibility of equilibria where $\lambda_1(1) < 1$. We now establish that: 1) such equilibria exist if and only if $2\pi_1\alpha > 1$; and 2) if this condition is satisfied, then there is a unique such equilibrium.

We know from above that in any equilibrium with $\lambda_1(1) < 1$, we have

$$\begin{cases} 1 > \lambda_1(1) > \lambda_1(0) > 0 \\ 1 = \lambda_2(1) > \lambda_2(0) > 0. \end{cases}$$

These inequalities imply that $X_1(\ell_1;\rho) > 0 = X_2(0;\rho)$ for each $\ell_1 = 0, 1$ and, therefore, that $\gamma_1(\ell_1, 0) = 1$. It follows that

$$\begin{cases} \Gamma_1(1) = 1 - 2\pi_2\gamma_2(1,1) \\ \Gamma_1(0) = 1 - 2\pi_2\gamma_2(0,1) \\ \Gamma_2(0) = 0. \end{cases}$$

Also, $1 > \lambda_1(1) > \lambda_1(0) > 0$ requires

$$\begin{cases} \frac{dEv_1}{d\lambda_1(1)} = 0 \Leftrightarrow \Gamma_1(0) = 1 - f_1 \\ \frac{dEv_1}{d\lambda_1(0)} = 0 \Leftrightarrow \Gamma_1(1) = 1 - f_1. \end{cases}$$

These two inequalities, together with the expressions for $\Gamma_1(0)$ and $\Gamma_1(1)$ obtained above, imply that $\gamma_2(1,1) = \gamma_2(0,1) = \frac{f_1}{2\pi_2} \in (0,1)$. In turn, the latter equalities imply $\Gamma_2(1) = f_1/2\pi_2$.

Since $\gamma_2(1,1) \in (0,1)$, it must be that $X_1(1;\rho)\alpha = X_2(1;\rho)$. Likewise, $\gamma_2(0,1) \in (0,1)$ requires $X_1(0;\rho)\alpha = X_2(1;\rho)$. Given that $X_1(0;\rho) = \beta_1^{Acc}(0)$, $X_1(1;\rho) = 1 - \beta_1^{Acc}(1)$ and $X_2(1;\rho) = 1/2$, we obtain from $X_1(0;\rho)\alpha = X_1(1;\rho)\alpha = X_2(1;\rho)$ that

$$\begin{cases} \lambda_1(1) = \frac{(2\pi_1\alpha - 1)(2\alpha - 1)}{4\alpha(\alpha-1)\pi_1} \\ \lambda_1(0) = \frac{4\pi_1\alpha - 1}{4\alpha(\alpha-1)(1-\pi_1)}. \end{cases}$$

Simple algebra establishes that $1 > \lambda_1(1) > \lambda_1(0) > 0$ if and only if $2\pi_1\alpha > 1$.

Using the expressions for $\lambda_1(1)$ and $\lambda_1(0)$ just obtained, we get $\beta_1^{Acc}(1) = \frac{2\alpha-1}{2\alpha} \in \left(\frac{1}{2}, 1\right)$ and $\beta_1^{Acc}(0) = \frac{1}{2\alpha} \in \left(0, \frac{1}{2}\right)$.

Finally, $\lambda_2(0) \in (0,1)$ requires

$$\frac{dEv_2}{d\lambda_2(0)} = 0 \Leftrightarrow (1 - \Gamma_2(1))\,\rho_2(1,0) = f_2.$$

Given the expression for $\Gamma_2(1)$ obtained above, we get $\rho_2(1,0) = \frac{2\pi_2 f_2}{2\pi_2 - f_1} \in (0,1)$. (Observe that $\Gamma_2(1) > \Gamma_2(0)$ and $\frac{dEv_2}{d\lambda_2(0)} = 0$ imply $\frac{dEv_2}{d\lambda_2(1)} > 0$, which is consistent with $\lambda_2(1) = 1$.)

Finally, we have $\beta_i(\ell_i, 1) = \rho_i(\ell_i, 1) = \theta_i$ for each $\ell_i = 0, 1$ and each $i = 1, 2$. ■

Proof of Proposition 4. We start by considering the case where $\frac{(1-\pi_1)f_1 + (1-\pi_2)f_2}{1-\pi_1\pi_2} > 1$. We know from part (1) of lemma 1 that an equilibrium with truthful lobbying exists when the PM has no subpoena power. We also know from part (1) of lemma 5 that in no equilibrium is the PM perfectly informed about θ when he has subpoena power. Hence $EU^{sub} < EU^{nosub} = \alpha + 1$.

We continue by considering the case where $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1\pi_2} \geq 1 \geq \frac{(1-\pi_1)f_1 + (1-\pi_2)f_2}{1-\pi_1\pi_2}$. We know from part (1) of lemma 1 and part (1) of lemma 5 that an equilibrium exists in which the PM gets perfectly informed about θ, whether the PM has subpoena power or not. Hence, $EU^{sub} = EU^{nosub} = \alpha + 1$.

It remains to consider the case where $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1\pi_2} < 1$. We know from part (2) of lemma 1 that a unique equilibrium exists in the game without subpoena power. In this equilibrium, the PM's ex ante expected payoff is given by

$$EU^{nosub} = (\alpha + 1) - \frac{2\pi_1\pi_2\alpha}{2\alpha - 1}.$$

We know from part (2a) of lemma 5 that similar overlobbying equilibria exist in the game with subpoena power. In each of these equilibria, the PM's ex ante expected payoff is equal to EU^{nosub}. We can infer that, in this case, $EU^{sub} \geq EU^{nosub}$. It remains to show that this inequality is actually an equality. This is necessarily the case if $2\pi_1\alpha \leq 1$ since part (2) of lemma 5 establishes that there is no other equilibrium in which $\delta_i > 0$ for $i = 1, 2$. If $2\pi_1\alpha > 1$, part (2b) of lemma 5 establishes that other equilibria exist in the game with subpoena power. In these equilibria, the PM's ex ante expected payoff is equal to

$$EU = (\alpha + 1) - \frac{\pi_1\alpha f_1 + 2\pi_2(\alpha - 1)}{2(\alpha - 1)}.$$

Since $2\pi_1\alpha > 1$ and $\pi_1 < 1/2$, simple algebra establishes that $EU < EU^{nosub}$. Hence we have $EU^{sub} = EU^{nosub}$.

This completes the proof since these three cases exhaust all possibilities. ■

The next lemma describes IGs' organization strategies in the game where the set of IGs is endogenous. Let η_i denote IG$_i$'s organization strategy, where $\eta_i \in [0,1]$ is the probability that IG$_i$ organizes.

Lemma 1. *IG$_1$'s organization strategy is given by*

$$\eta_1 \begin{cases} = 1 & \text{if } \pi_1(1 - f_1) > c_1 \\ \in [0,1] & \text{if } \pi_1(1 - f_1) = c_1 \\ = 0 & \text{if } \pi_1(1 - f_1) < c_1. \end{cases}$$

IG$_2$'s organization strategy is as follows:

(1) *When* $N=2$,

$$\eta_2\begin{cases}=1 & \text{if } \pi_2(1-f_2)>c_2\\ \in[0,1] & \text{if } \pi_2(1-f_2)=c_2\\ =0 & \text{if } \pi_2(1-f_2)<c_2\end{cases}$$

if $\frac{\pi_1 f_1+\pi_2 f_2}{\pi_1\pi_2}\geq 1$, *and*

$$\eta_2\begin{cases}=1 & \text{if } \pi_2\left\{1-\eta_1\pi_1\left[\frac{2\alpha+(\alpha-1)\frac{f_1}{\pi_2}}{2\alpha-1}-\frac{\pi_1 f_1+\pi_2 f_2}{\pi_1\pi_2}\right]-f_2\right\}>c_2\\ \in[0,1] & \text{if } \pi_2\left\{1-\eta_1\pi_1\left[\frac{2\alpha+(\alpha-1)\frac{f_1}{\pi_2}}{2\alpha-1}-\frac{\pi_1 f_1+\pi_2 f_2}{\pi_1\pi_2}\right]-f_2\right\}=c_2\\ =0 & \text{if } \pi_2\left\{1-\eta_1\pi_1\left[\frac{2\alpha+(\alpha-1)\frac{f_1}{\pi_2}}{2\alpha-1}-\frac{\pi_1 f_1+\pi_2 f_2}{\pi_1\pi_2}\right]-f_2\right\}<c_2\end{cases}$$

if $\frac{\pi_1 f_1+\pi_2 f_2}{\pi_1\pi_2}<1$.

(2) *When* $N=1$,

$$\eta_2\begin{cases}=1 & \text{if } \pi_2\left[1-\eta_1(1-f_2)-f_2\right]>c_2\\ \in[0,1] & \text{if } \pi_2\left[1-\eta_1(1-f_2)-f_2\right]=c_2\\ =0 & \text{if } \pi_2\left[1-\eta_1(1-f_2)-f_2\right]<c_2\end{cases}$$

if $f_2>1-\pi_1$, *and*

$$\eta_2\begin{cases}=1 & \text{if } \pi_2\left[1-\eta_1\pi_1-f_2\right]>c_2\\ \in[0,1] & \text{if } \pi_2\left[1-\eta_1\pi_1-f_2\right]=c_2\\ =0 & \text{if } \pi_2\left[1-\eta_1\pi_1-f_2\right]<c_2\end{cases}$$

if $f_2\leq 1-\pi_1$.

Proof of Lemma 6. Observe that if IG_i does not organize, the PM's belief about θ_i is given by $\pi_i<1/2$. In this case, the PM chooses $p_i=0$, and IG_i's expected payoff is $Ev_i(n_i=0)=0$, where $Ev_i(n_i)$ denotes IG_i's equilibrium expected payoff given its organization decision $n_i\in\{0,1\}$.

We know from the proof of proposition 2 that IG_1's expected payoff if it organizes is given by

$$Ev_1(n_1=1)=\pi_1(1-f_1)-c_1.$$

Consider now IG_2's expected payoff when it organizes. There are three cases to consider:

(1) $N=1$ and $f_2>1-\pi_1$. If IG_1 is organized (which occurs with probability η_1), IG_2 will abstain from lobbying. The PM will then believe $\theta_2=1$ with probability $\beta_2^{Acc}(0)=\pi_2<1/2$ and will then choose $p_2=0$. In this case, IG_2's expected payoff is equal to zero. If IG_1 is not organized, IG_2 will lobby truthfully and the PM will choose $p_2=\theta_2$. In this case, IG_2's expected payoff is equal to $\pi_2(1-f_2)$. To sum up, IG_2's expected payoff if it organizes is here given by

$$Ev_2(n_2=1)=(1-\eta_1)\pi_2(1-f_2)-c_2.$$

(2) Either $N=1$ and $f_2\leq 1-\pi_1$, or $N=2$ and $\frac{\pi_1 f_1+\pi_2 f_2}{\pi_1\pi_2}\geq 1$. In either of these two cases, any organized IG lobbies truthfully (by lemma 1 and lemma 2) and the PM chooses $p_2=\theta_2$, unless the two IGs are organized

and $\theta = (1, 1)$, in which case the PM chooses $p = (1, 0)$. IG$_2$'s expected payoff if it organizes is here given by

$$Ev_2\left(n_2 = 1\right) = \begin{cases} \pi_2\left(1 - f_2\right) - c_2 & \text{if } N = 2 \\ \pi_2\left[\eta_1\left(1 - \pi_1\right) + \left(1 - \eta_1\right) - f_2\right] - c_2 & \text{if } N = 1. \end{cases}$$

(3) $N = 2$ and $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$. If IG$_1$ is not organized, IG$_2$ will lobby truthfully. In this case, IG$_2$'s expected payoff is equal to $\pi_2\left(1 - f_2\right)$. If IG$_1$ is organized, both IGs will overlobby (by lemma 1). We know from the proof of proposition 2 that, in this case, IG$_2$'s expected payoff is equal to

$$\pi_2\left\{1 - \pi_1\left[\frac{\alpha\left(2\pi_2 - f_1\right) - \frac{\pi_2}{\pi_1}\left(2\alpha - 1\right) f_2}{\pi_2\left(2\alpha - 1\right)}\right] - f_2\right\}.$$

To sum up, IG$_2$'s expected payoff if it organizes is here given by

$$Ev_2\left(n_2 = 1\right) = \pi_2\left\{1 - \eta_1\pi_1\left[\frac{\alpha\left(2\pi_2 - f_1\right) - \frac{\pi_2}{\pi_1}\left(2\alpha - 1\right) f_2}{\pi_2\left(2\alpha - 1\right)}\right] - f_2\right\} - c_2.$$

Thus, IG$_i$ organizes if $Ev_i\left(n_i = 1\right) > Ev_i\left(n_i = 0\right) = 0$, and only if $Ev_i\left(n_i = 1\right) \geq Ev_i\left(n_i = 0\right) = 0$. ■

We can make three observations relative to lemma 6.

First, $\pi_i < 1/2$ implies that if it does not organize, IG$_i$ gets $p_i = 0$. Moreover, an IG lobbies truthfully in the equilibrium of the subgame where it is the only organized IG. This means that the difference in organization incentives between the $N = 1$-game and the $N = 2$-game is associated with the subgame where the two IGs are organized.

Second, IG$_1$'s organization strategy does not depend on N. This is because the PM prioritizes issue 1 when IG$_1$ is organized and lobbies. When $\theta_1 = 1$, IG$_1$ lobbies and gets $p_1 = 1$ with probability one. When $\theta_1 = 0$, IG$_1$ does not lobby or it randomizes between lobbying and not lobbying; in either case IG$_1$ gets zero expected payoff. Thus, whether $\theta_1 = 1$ or $\theta_1 = 0$, IG$_1$'s expected payoff, and therefore its organization strategy, is independent of N.

Third, IG$_2$'s organization strategy depends on N (if and only if IG$_1$ organizes with positive probability). To understand why, we partition the parameter space into the same three regions as in proposition 1 and consider subgames where IG$_1$ is organized (since, as argued in observation 1 above, the differences between $N = 1$ and $N = 2$ occur only in the subgame where the two IGs are organized).

(1) $f_2 > 1 - \pi_1$: When $N = 1$, IG$_2$ does not want to organize. This is because it anticipates that IG$_1$ will lobby truthfully and, then, that it, IG$_2$, will not lobby at all. IG$_2$'s expected payoff will then be equal to zero. When $N = 2$, IG$_2$ anticipates truthful lobbying and positive expected payoff. Thus, IG$_2$ is (weakly) more likely to organize when $N = 2$ than when $N = 1$.

(2) $f_2 \in \left[\pi_1\left(1 - \frac{f_1}{\pi_2}\right), 1 - \pi_1\right]$: IG$_2$ anticipates truthful lobbying whether $N = 1$ or $N = 2$. In this case, IG$_2$'s expected payoff is bigger when $N = 2$ than when $N = 1$ since IG$_2$ does not have to bear the agenda constraint cost when $N = 2$, in contrast to when $N = 1$. It follows that, as in the first region, IG$_2$ is more likely to organize when $N = 2$ than when $N = 1$.

(3) $f_2 < \pi_1 \left(1 - \frac{f_1}{\pi_2}\right)$: In contrast to what happens in the other two regions of the parameter space, here IG$_2$ can be more likely to organize when $N = 1$ than when $N = 2$. This happens when, as discussed in section 5.3, the overlobbying externality cost exceeds the agenda constraint cost. In that case, IG$_2$'s expected payoff is bigger when $N = 1$ than when $N = 2$.

Proof of Proposition 5. We start by establishing the sufficiency of the three conditions in the statement.

Condition (1) implies that $\eta_1^{N=1} = \eta_1^{N=2} = 1$ (lemma 6), and IG$_1$'s expected payoff is given by $EU_1^{N=1} = EU_1^{N=2} = \pi_1 (1 - f_1) - c_1$.

Together, condition (3) and $\alpha > 1$ imply $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$, and case (3) in lemma 6 then applies. When $N = 1$, condition (2) implies $\eta_2^{N=1} = 1$ and $EU_2^{N=1} = \pi_2 (1 - \pi_1 - f_2) - c_2 > 0$. Moreover, $\eta_1^{N=1} = \eta_1^{N=2} = 1$ implies $EU_{PM}^{N=1} = \alpha + (1 - \pi_1 \pi_2)$. When $N = 2$, two cases are possible:

(1) IG$_2$ organizes. In this case, condition (3) implies $EU_2^{N=1} \geq EU_2^{N=2}$. Moreover, given the equilibrium strategies (case (2) in lemma 1), we get $EU_{PM}^{N=2} = (\alpha + 1) - \frac{2\pi_1 \pi_2 \alpha}{2\alpha - 1} < EU_{PM}^{N=1}$.
(2) IG$_2$ does not organize. In this case, $EU_2^{N=2} = 0 < EU_2^{N=1}$. Moreover, the PM gets informed on issue 1 and, given $\pi_2 < 1/2$, chooses $p = (\theta_1, 0)$. The PM's expected payoff is then given by $EU_{PM}^{N=2} = \alpha + (1 - \pi_2) < EU_{PM}^{N=1}$.

To sum up, we have $EU_1^{N=1} = EU_1^{N=2}$, $EU_2^{N=1} \geq EU_2^{N=2}$ and $EU_{PM}^{N=1} > EU_{PM}^{N=2}$.

We now establish the necessity of each of the three conditions in the statement. Suppose $EU_k^{N=1} \geq EU_k^{N=2}$ for each player $k \in \{1, 2, PM\}$, with at least one inequality strict.

First, it must be that $c_1 \leq \pi_1 (1 - f_1)$, so that $\eta_1^N > 0$ (by lemma 6). To see this, assume by way of contradiction that $c_1 > \pi_1 (1 - f_1)$. We then have $\eta_1^N = 0$ for each $N \in \{1, 2\}$. It follows that $EU_1^{N=1} = EU_1^{N=2} = 0$. Moreover, we know from lemma 6 that $EU_2^{N=1} = EU_2^{N=2}$. Finally, the PM chooses $p_1 = 0$, implying that an agenda constraint will not be binding and, therefore, that $EU_{PM}^{N=1} = EU_{PM}^{N=2}$. Hence $EU_k^{N=1} = EU_k^{N=2}$ for every player $k \in \{1, 2, PM\}$, a contradiction.

Second, it must be that $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$. This is because otherwise case (1) or case (2) of lemma 6 would apply, and IG$_2$ would have the strongest incentives to organize when $N = 2$. If IG$_2$ were to organize when $N = 2$, we would then have $EU_2^{N=2} > EU_2^{N=1}$, a contradiction. If IG$_2$ were to not organize when $N = 2$, it would not organize either when $N = 1$, and we would have $EU_k^{N=2} = EU_k^{N=1}$ for each player $k \in \{1, 2, PM\}$, a contradiction.

Third, it must be that $\frac{\alpha \pi_1 f_1 + (2\alpha - 1)\pi_2 f_2}{\pi_1 \pi_2} \leq 1$. The argument is the same as in the proof of proposition 2. (Observe that this restriction implies $\frac{\pi_1 f_1 + \pi_2 f_2}{\pi_1 \pi_2} < 1$.)

Finally, it must be that $c_2 \leq \pi_2 (1 - \pi_1 - f_2)$, so that $\eta_2^{N=1} > 0$ (by lemma 6). To see this, assume by way of contradiction that $c_2 > \pi_2 (1 - \pi_1 - f_2)$. We would then have $\eta_2^{N=1} = 0$ and, as mentioned above, $\eta_2^{N=2} = 0$. We would have again that $EU_k^{N=2} = EU_k^{N=1}$ for every player $k \in \{1, 2, PM\}$, a contradiction. ■

Details of Example 1

In this section we present the main qualitative results of our model by means of an illustrative example. In particular, we show that, for a range of parameters, an agenda constraint results in better information transmission in equilibrium. We also show that the introduction of an agenda constraint can generate a Pareto improvement, the equilibrium outcome of the game with agenda constraint ($N = 1$) Pareto-dominating the equilibrium outcome of the game without agenda constraint ($N = 2$).

We start by characterizing equilibrium access and policy choice strategies in the subgame following truthful lobbying. We say that lobbying is truthful if each IG lobbies when and only when it has favorable evidence ($\theta_i = 1$); formally, $\lambda_i(0) = 0$ and $\lambda_i(1) = 1$ for each $i = 1, 2$.[1] Consistency requires that the beliefs of the PM at the access stage must be $\beta_i^{Acc}(0) = 0$ and $\beta_i^{Acc}(1) = 1$ for each $i = 1, 2$, i.e., the PM must believe that an IG has favorable information if and only if that IG lobbies. Under these beliefs, any feasible access strategy (γ_1, γ_2) is optimal since there is no further information to be gained; through lobbying decisions, the PM is already perfectly informed about θ.

Let's look at the optimal policy choice under truthful lobbying. When IG$_i$ does not lobby, the optimal policy choice is straightforward: keep the status quo on issue i, i.e., $p_i = 0$. When only IG$_i$ lobbies, the policy choice is $p_i = 1$ if and only if IG$_i$ reveals $\theta_i = 1$. When both IGs lobby, the PM's interim beliefs imply that, absent further information, he gets a positive payoff from implementing reform on either issue. If there is no agenda constraint ($N = 2$), he chooses $p = (1, 1)$, unless he grants access to IG$_i$ and finds $\theta_i = 0$. If there is an agenda constraint ($N = 1$), the PM will "prioritize" reform on issue 1 since this issue is more important to him ($\alpha > 1$), i.e., he will implement reform on issue 1 ($p = (1, 0)$), unless he grants access to IG$_1$ and finds $\theta_1 = 0$.[2] Hence, the PM's optimal policy strategy when *both* IGs lobby can be summarized as follows:

: CASE 1: $N = 2$. Implement reform on **both issues** unless IG$_i$ was granted access and revealed $\theta_i = 0$, in which case implement reform only on issue $-i$.

: CASE 2: $N = 1$. Implement reform on **issue 1** unless IG$_1$ was granted access and revealed $\theta_1 = 0$, in which case implement reform on issue 2.

We continue by considering a specific numerical example. Let parameters take the following values:

- $\alpha = 2$, i.e., the PM finds issue 1 twice as important as issue 2;
- $\pi_1 = \pi_2 = 2/5$, i.e., the PM is ex ante biased against reforms; and
- the lobbying cost for each IG is $f = 1/20$.

Game without agenda constraint ($N = 2$). In this section, we characterize the set of equilibria for the game without agenda constraint ($N = 2$).

We start by showing that there is no equilibrium in which the PM is always perfectly informed about θ. To see this, assume by way of contradiction that such an equilibrium were to exist. In this case lobbying must be truthful, as we show

[1] The opposite strategies, i.e., lobby if and only if it has unfavorable information cannot be part of an equilibrium since lobbying is costly.

[2] This does not happen on the equilibrium path but is part of the description of the optimal strategy of the PM.

in section 5.[3] Let $\widehat{\gamma}_i \in [0,1]$ denote the equilibrium probability with which IG$_i$ is granted access when both IGs lobby, with the restriction that $\widehat{\gamma}_1+\widehat{\gamma}_2 = 1$. Given the optimal policy choice strategy as described above, an IG which lobbies and is not granted access gets its reform adopted. IG$_i$'s expected policy payoff from lobbying when $\theta_i = 0$ is then equal to $2/5 \cdot \widehat{\gamma}_{-i}$, which corresponds to the probability that IG$_{-i}$ lobbies and is granted access. For IG$_i$ to not deviate and lobby when $\theta_i = 0$, it must be that $2/5 \cdot \widehat{\gamma}_{-i} \leq 1/20$, i.e., the expected policy gain from lobbying must not exceed the lobbying cost. This inequality is satisfied for each $i = 1, 2$ only if $\widehat{\gamma}_i \leq 1/8$ for $i = 1, 2$, which contradicts $\widehat{\gamma}_1 + \widehat{\gamma}_2 = 1$. Thus, in equilibrium the PM does not get fully informed about θ.

We continue by characterizing the equilibrium set for the game without agenda constraint. As we show in the next section, the equilibrium of this game is unique and corresponds to the following strategies and beliefs:

(1) The lobbying strategies are given by $\lambda_i(1) = 1$ for each $i = 1, 2$, $\lambda_1(0) = 2/9$ and $\lambda_2(0) = 2/3$, i.e., IG$_i$ always lobbies when it has favorable information and randomizes between lobbying and not lobbying when it has unfavorable information.
(2) The access strategy is such that when both IGs lobby, $\gamma_1(1,1) = 15/16$ and $\gamma_2(1,1) = 1/16$, i.e., the PM gives access priority to IG$_1$. When only one IG lobbies, it is granted access with probability one. When neither IG lobbies, the PM cannot grant access to any IG.
(3) The policy strategy is such that the PM chooses $p_1 = 1$ when IG$_1$ lobbies and is not granted access. The PM chooses $p_2 = 1$ with probability 1/10 when IG$_2$ lobbies and is not granted access. Finally, $p_i = 0$ when IG$_i$ does not lobby, and $p_i = \theta_i$ when IG$_i$ lobbies and is granted access.
(4) PM's beliefs at the access stage are obtained from the lobbying strategies using Bayes' rule: $\beta_i^{Acc}(0) = 0$ for each $i = 1, 2$, $\beta_1^{Acc}(1) = \frac{2/5}{2/5+(3/5)\cdot(2/9)} = 3/4$ and $\beta_2^{Acc}(1) = \frac{2/5}{2/5+(3/5)\cdot(2/3)} = 1/2$.

We now verify that these strategies and beliefs constitute an equilibrium.

First, it is clear from the description of the policy stage in section 3 that the policy strategy maximizes the PM's expected payoff. The exact randomization on p_2 when IG$_2$ lobbies and is not granted access justifies IG$_2$'s randomization over its lobbying decision when $\theta_2 = 0$.

Second, consider the PM's access decision when both IGs lobby. The PM believes $\theta_1 = 1$ with probability 3/4 and $\theta_2 = 1$ with probability 1/2.

(1) If the PM grants access to IG$_1$, he learns θ_1 and chooses the correct p_1, while randomizing on p_2 and choosing the correct p_2 with probability 1/2. The PM's expected payoff is equal to $2 + 1/2 = 5/2$.

[3] Intuitively, both IGs must be lobbying with (sufficiently high) positive probability. Moreover, at least one of the two IGs must be lobbying truthfully since the PM can grant access to only one IG, thereby requiring lobbying decisions to be perfectly informative for at least one issue. If only one IG lobbies truthfully, the PM will necessarily choose to grant access to the 'untruthful' IG when this IG lobbies since the value of the information obtained by granting access to the 'untruthful' IG is greater than the value of the information obtained by granting access to the 'truthful' IG (where no information is to be gained). This implies that the 'truthful' IG has an incentive to deviate and lobby when it has unfavorable evidence, since it has a sufficiently high probability of not being awarded access and, therefore, of not having to reveal its information and getting its reform adopted.

(2) If the PM grants access to IG_2, he learns θ_2 and chooses the correct p_2, while choosing $p_1 = 1$, which is the correct choice with probability 3/4. The PM's expected payoff is equal to $(3/4)\ \ 2 + 1 = 5/2$.

Thus, when the two IGs lobby, the PM is indifferent between granting access to IG_1 and granting access to IG_2. The exact randomization justifies IG_1's lobbying randomization when $\theta_1 = 0$.

Third, consider IGs' lobbying strategies. We start by observing that if IG_i does not lobby, the PM believes $\theta_i = 0$ and chooses $p_i = 0$. IG_i's payoff is then equal to zero.

We continue by checking that IG_1's lobbying strategy is an equilibrium strategy.

(1) When $\theta_1 = 1$, IG_1 is strictly better off lobbying. If this IG lobbies, the PM adopts $p_1 = 1$, independently of whether or not he awards access to IG_1. IG_1's payoff is thus equal to $1 - 1/20 = 19/20 > 0$, which is strictly bigger than if IG_1 were not lobbying.
(2) When $\theta_1 = 0$, IG_1 is indifferent between lobbying and not lobbying. If IG_1 lobbies, the PM chooses $p_1 = 1$ if and only if he does not grant access to IG_1; otherwise, IG_1 must reveal $\theta_1 = 0$ and the PM chooses $p_1 = 0$. This event happens if and only if IG_2 lobbies and is the one to be granted access, which occurs with probability $[2/5+(3/5)\cdot(2/3)]\cdot(1/16)=1/20$. IG_1's expected payoff is thus equal to $\frac{1}{20} - \frac{1}{20} = 0$, which corresponds to the probability $p_1 = 1$ minus the lobbying cost. Thus, IG_1 gets zero expected payoff whether it lobbies or not. The exact randomization justifies the PM's access randomization when both IGs lobby.

It remains to check that IG_2's lobbying strategy is an equilibrium strategy.

(1) When $\theta_2 = 1$, IG_2 is strictly better off lobbying. If this IG lobbies, the PM adopts $p_2 = 1$ with probability 11/20 (viz. with probability 1 if he grants access to IG_2 and with probability 1/10 if IG_1 lobbies and is granted access). IG_2's expected payoff is equal to $\frac{11}{20} - \frac{1}{20} = 1/2 > 0$, which is strictly bigger than if IG_2 were not lobbying (which would be equal to zero).
(2) When $\theta_2 = 0$, IG_2 is indifferent between lobbying and not lobbying. If this IG lobbies, the PM chooses $p_2 = 1$ if and only if he awards access to IG_1 and randomizes in favor of $p_2 = 1$. This event happens with probability $[2/5+(3/5)\cdot(2/9)]\cdot(15/16)\cdot(1/10)=1/20$, i.e., the probability that IG_1 lobbies, is granted access, and the PM chooses $p_2 = 1$ when IG_2 lobbies but is not granted access. IG_2's expected payoff from lobbying is equal to $\frac{1}{20} - \frac{1}{20} = 0$. Thus, IG_2's expected payoff is equal to zero whether it lobbies or not. The exact randomization justifies the PM's policy randomization over p_2 when IG_2 lobbies but is not granted access.

Equilibrium ex ante expected payoffs of the three players are equal to

$$\begin{aligned}
Ev_1^{N=2} = &\quad \pi\ \ (1-f) + (1-\pi)\ \ 0 = \tfrac{19}{50} \\
Ev_2^{N=2} = &\quad \pi\left\{1 - \left[\pi + (1-\pi)\cdot\tfrac{2}{9}\right]\cdot\left(\tfrac{15}{16}\right)\cdot\left(\tfrac{9}{10}\right) - f\right\} + (1-\pi)\cdot 0 = \tfrac{1}{5} \\
EU^{N=2} = &\quad \alpha\left\{1 - (1-\pi)\cdot\tfrac{2}{9}\cdot\left[\pi + (1-\pi)\cdot\tfrac{2}{3}\right]\cdot\tfrac{1}{16}\right\} \\
&\quad + 1 - \left\{\left[\pi + (1-\pi)\cdot\tfrac{2}{9}\right]\cdot\tfrac{15}{16}\cdot\left[\pi\cdot\tfrac{9}{10} + (1-\pi)\cdot\tfrac{2}{3}\cdot\tfrac{1}{10}\right]\right\} = \tfrac{209}{75},
\end{aligned}$$

for IG_1, IG_2 and the PM, respectively.

Game with agenda constraint ($N = 1$). In this section, we characterize the set of equilibrium outcomes for the game with agenda constraint ($N = 1$). In particular, we show that the PM gets perfectly informed about θ.

Consider the access strategy where the PM grants access to IG_1 when both IGs lobby, i.e., $\gamma_1(1,1) = 1$. This access strategy, the policy choice strategy described above, and truthful lobbying strategies are part of an equilibrium. Moreover, as we show in section 5, *any* equilibrium in this parameter region has truthful lobbying, and all equilibria are outcome and payoff equivalent. This contrasts with the absence of an equilibrium of the $N = 2-$game where the PM always gets perfectly informed about θ. This illustrates our result that an agenda constraint can lead to better information transmission (Proposition 1).

To show that these strategies are part of an equilibrium, it remains only to establish that truthful lobbying is an equilibrium strategy. Observe that issue i never gets reformed when $\theta_i = 0$, implying that IG_i has no incentive to deviate and lobby in this state. When $\theta_1 = 1$, IG_1 gets payoff $1 - 1/20 = 19/20$ from lobbying (i.e., it gets $p_1 = 1$ and must bear lobbying cost $f = 1/20$). When $\theta_2 = 1$, IG_2 gets expected payoff $3/5 - 1/20 = 11/20$ from lobbying (i.e., it gets $p_2 = 1$ when $\theta_1 = 0$, which happens with probability $3/5$, and must bear lobbying cost $f = 1/20$). Since each IG gets zero payoff if it does not lobby ($p_i = 0$ since $\beta_i^{Acc}(0) = 0$), neither IG_i wants to deviate and not lobby when $\theta_i = 1$.

To sum up, through lobbying decisions the PM gets perfectly informed about θ. He always chooses the correct p_1. He also chooses the correct p_2 unless $\theta = (1,1)$, in which case the agenda constraint is binding and the PM reforms issue 1, while keeping the status quo for issue 2. Thus, equilibrium expected payoffs are $EU^{N=1} = \alpha + 1 - \pi^2 = 71/25$ for the PM, $Ev_1^{N=1} = \pi \cdot (1 - f) = 19/50$ for IG_1,and $Ev_2^{N=1} = \pi \cdot (1 - \pi - f) = 11/50$ for IG_2.

Pareto improvement. Comparing equilibrium expected payoffs in the two games, we get

$$\begin{aligned}
Ev_1^{N=2} &= \frac{19}{50} = Ev_1^{N=1} \\
Ev_2^{N=2} &= \frac{1}{5} < \frac{11}{50} = Ev_2^{N=1} \\
EU^{N=2} &= \frac{209}{75} < \frac{71}{25} = EU^{N=1}.
\end{aligned}$$

Thus, IG_1 is ex ante as well off in the $N = 1$-game as in the $N = 2$-game, while IG_2 and the PM are each ex ante strictly better off in the former than in the latter. This illustrates our second result that, from an ex ante point of view, the introduction of an agenda constraint can generate a Pareto improvement (Proposition 2).

To understand this result, observe that the introduction of an agenda constraint has a depressing effect on the PM's expected payoff by preventing him from reforming both issues. For the introduction of an agenda constraint to increase the PM's expected payoff, it must then be that the PM gets better informed about θ with than without agenda constraint. This is made possible by the fact that the agenda constraint allows the PM to use his access strategy to 'discipline' the lobbying behavior of IGs, something he cannot do without agenda constraint. More specifically,

(1) in the $N = 1$-game, the PM can proceed 'lexicographically', prioritizing issue 1 by awarding access to IG_1 whenever it lobbies, and adopting $p = (1, 0)$ if and only if IG_1 reveals $\theta_1 = 1$. This strategy induces both IGs to lobby truthfully: IG_1 because it knows it will be granted access if it lobbies; IG_2 because it knows its lobbying decision will matter for the policy outcome if and only if $\theta_1 = 0$, in which case IG_1 will not lobby and IG_2 will necessarily be granted access if it lobbies.

(2) in the $N = 2$-game, the PM can no longer 'discipline' IGs by prioritizing issue 1. Even if the PM were to prioritize issue 1, IG_2's lobbying decision would still matter since the PM can reform both issues. It follows that if IGs were to lobby truthfully, when IG_2 lobbies and is not granted access, the PM would believe $\theta_2 = 1$ and would choose $p_2 = 1$. If lobbying is not too costly, as it is the case in this example, IG_2 would then want to deviate and lobby when $\theta_2 = 0$, hoping that it will not be granted access. In other words, the fact that the PM can reform both issues while he can grant access to only one IG creates an incentive for IGs to overlobby. We say that IG_i overlobbies if it lobbies more often than it would if it were to lobby truthfully (viz. $\lambda_i(1) = 1$ and $\lambda_i(0) > 0$). In equilibrium IGs overlobby up to the point where they are indifferent between lobbying and not lobbying when they have unfavorable information, i.e., up to the point where, in expectation, all the rent from overlobbying is exhausted and the expected payoff in state $\theta_i = 0$ is equal to zero whether IG_i lobbies or not.

IG_1 gets the same expected payoff in both games. This is because the PM prioritizes issue 1 in the $N = 1$- game and IG_1 overlobbies in the $N = 2$-game.

IG_2 gets a higher expected payoff in the $N = 1$-game than in the $N = 2$-game. To see this, observe that when $\theta_2 = 0$, IG_2 gets zero expected payoff in both games. This is because IG_2 lobbies truthfully in the $N = 1$-game and exhausts, in expectation, the rent from overlobbying in the $N = 2$-game. When $\theta_2 = 1$, IG_2 benefits from the relaxation of the agenda constraint: in the $N = 2$-game, IG_2 can get its reform adopted even when $\theta_1 = 1$, which is not possible in the $N = 1$-game since the PM prioritizes issue 1. At the same time, IG_2's overlobbying in state $\theta_2 = 0$ generates a negative externality on IG_2's $\theta_2 = 1$-self, by undermining the PM's belief that $\theta_2 = 1$ when IG_2 lobbies but is not granted access. The latter induces the PM to adopt $p_2 = 1$ with probability less than one. Given the parameter values in this example, the overlobbying externality cost exceeds the benefit from the relaxation of the agenda constraint, implying that IG_2 is ex ante strictly better off in the $N = 1$-game than in the $N = 2$-game.

Finally, the PM gets a higher expected payoff in the $N = 1$-game than in the $N = 2$-game. On the one hand, the PM benefits from the relaxation of the agenda constraint by being able to reform both issues. On the other hand, overlobbying implies that the PM is lesser informed in the $N = 2$-game than in the $N = 1$-game. For the parameter values in this example, the informational benefit from the introduction of the agenda constraint exceeds the cost from the agenda constraint.