THE INVERSE PROBLEM OF DETERMINING THE FILTRATION FUNCTION AND PERMEABILITY REDUCTION IN FLOW OF WATER WITH PARTICLES IN POROUS MEDIA

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ABSTRACT. Deep bed filtration of particle suspensions in porous media occurs during water injection into oil reservoirs, drilling fluid invasion of reservoir production zones, fines migration in oil fields, bacteria, viruses or contaminant transport in groundwater, industrial filtering, etc. The basic features of the process are particle capture by the porous medium and consequent permeability reduction.

Models for deep bed filtration contain two coefficients that represent rock and fluid properties: the filtration function, which is the fraction of captured particles per unit of particle path length, and formation damage function, which is the ratio between reduced and initial permeabilities. The coefficients cannot be measured directly in the laboratory or in the field; therefore, they must be calculated indirectly by solving inverse problems. The practical petroleum and environmental engineering purpose is to predict injectivity loss and particle penetration depth around wells Reliable prediction requires precise knowledge of these two coefficients.

In this work we determine these coefficients from pressure drop and effluent concentration histories, measured in one-dimensional laboratory experiments. The filtration function is recovered by optimizing a nonlinear functional with box constraints. The permeability reduction is recovered likewise, taking into account the filtration function already found. The recovery method consists of optimizing Tikhonov's functionals in appropriate subdomains. In both cases, the functionals are derived from least square formulations of the deviation between experimental data and quantities predicted by the model.

Deep bed filtration, Suspension transport, Porous media, Inverse problem, Tikhonov regularization, Formation damage, System of convection-reaction equations

1. INTRODUCTION

Severe injectivity decline during sea- or produced water injection is a serious problem in offshore waterflood projects. This decline results from permeability impairment because the rock captures particles from injected water. Reliable modelling-based prediction of injectivity decline is important for the design of injected water treatment or management by water filtering, injection of combinations of sea- or produced water, etc.

Formation damage induced by penetration of drilling fluid into a reservoir also reults from suspended particle capture by rocks and consequent permeability reduction. Other petroleum applications for which formation damage is important include sand production control, fines migration and deep bed filtration in gravel packs. Deep bed filtration also occurs in industrial water filtering, in propagation of contaminants (incluing viruses, bacteria, etc) through aquifers, and in other environmental processes.

Mathematical models for filtration processes contain two parametric functions describing properties of the aqueous suspension and of the porous medium: the filtration function, i.e., the probability for a particle to be captured per unit particle path length, and the permeability reduction, i.e., the ratio between the reduced and the initial permeabilities. Particle

deposition alters pore space geometry and hydraulic resistivity; in turn, pore geometry alters conditions for further deposition, so it is natural to take the deposited concentration as the basic independent variable for the filtration and formation damage functions.

The filtration and formation damage functions cannot be measured directly, rather they must be recovered indirectly from experimental measurements by solving inverse problems for deep bed filtration. In one-dimensional laboratory flow experiments, it is possible to measure the time series of suspended particle effluent concentration and of pressure drop. Methods for using the effluent concentration and pressure drop histories to determine constant filtration coefficient and one-parameter permeability reduction function were presented in [17], [20], [3] and [4]. In this work, we present a more flexible method using parameter optimization.

Moreover, we introduce regularization to generalize the procedure for a two-parameter family of functions given in [11]. The recovery procedure for the filtration and permeability reduction functions presented in this work consists of optimizing functionals using the projection gradient method with box constraints developed in [6]. The functionals to be minimized are obtained from a least squares formulation taking into account the difference between experimental data and quantities predicted by the model. The box constraints reflect physical properties of the solution such as positivity and monotonicity.

This paper is organized as follows. In Section 2, we present the deep bed filtration model with formation damage as a system of two hyperbolic equations for suspended and retained concentrations and a pressure balance equation. We also explain how to solve this system by integrating two families of ordinary differential equations. In Section 3, we describe optimization procedures to solve two inverse problems. In the first one, we use the the calculated effluent concentration history to define the functional that is minimized to determine the filtration function. In the second one, we use the calculated deposition to find the pressure drop history and define the functional that is minimized to determine the permeability reduction function. In Section 4, we validate the recovery methods an and we examine the

sensitivity of these inverse problems by means of synthetic data. In Section 5, we apply the method to experimental data and discuss the recovered functions as obtained by solving the two inverse problems.

2. FLOW OF WATER WITH PARTICLES IN POROUS MEDIA.

In this section we present the physical model for the flow of water with suspended particles suffering retention in porous media. This model was developed in [3] based on [12]. During injection, the suspended particles are gradually retained, reducing the permeability of the medium. This phenomenon, called *deep bed filtration with formation damage*, is modelled by the system of equations:

$$\frac{\partial}{\partial t}\left(\phi c + \sigma\right) + U\frac{\partial c}{\partial x} = 0, \tag{2.1}$$

$$\frac{\partial \sigma}{\partial t} = \lambda(\sigma)Uc, \tag{2.2}$$

$$U = -\frac{k_0\, k(\sigma)}{\mu}\,\frac{\partial p}{\partial x}. \tag{2.3}$$

Here the concentrations of dispersed and deposited particles are $c(x,t) \in [0,1]$ and $\sigma(x,t) \in [0,\phi]$, where ϕ is a dimensionless quantity between 0 and 1, called the *rock porosity*: it is the

fraction of the rock volume available to the fluid. Eq. (2.2) can only be valid if $\sigma \ll \phi$ holds. The dependence of the retention rate on σ is expressed by $\lambda(\sigma)$, which is called the *filtration function*. The physical domain is $t > 0$ and $0 \le x \le L$, where L is the length of the core.

We assume that permeability reduction $k(\sigma)$ is due to particle retention, and that it is a decreasing function of the retained concentration. Eq. (2.3) is a form of Darcy's law relating the flow rate U to the pressure p. Here, k_0 is the absolute rock permeability and $k(\sigma)$ is the permeability reduction due to the retained particles σ; when expressed as a function of σ, it is called the *formation damage function*. It is normalized so that $k(0) = 1$, i.e. it is one for clean porous rock. In general, the water viscosity μ can be considered constant for small particle concentrations.

2.1. Boundary and measured data.
As initial data, we assume that the rock is clean and contains water with no particles; as boundary data, we assume that the solid particle concentration entering the porous medium is given, and that the effluent concentration is measured:

$$\sigma(x,0) = 0 \quad \text{and} \quad c(x,0) = 0, \tag{2.4}$$

$$c(0,t) = c_i(t) > 0, \quad t > 0, \tag{2.5}$$

$$c(L,t) = c_{exp}(t) > 0, \quad t > 0. \tag{2.6}$$

The pressure drop $\Delta p_{exp} = p(L,t) - p(0,t)$ is also measured in laboratory experiments. The quantity $\sigma(0,t)$, however, needs to be determined by the model. Along the line $x = 0$, we obtain from Eqs. (2.2) and (2.5):

$$\frac{d\sigma(0,t)}{dt} = \lambda(\sigma(0,t))U c_i(t), \quad \text{and} \quad \sigma(0,0) = 0, \tag{2.7}$$

where U is given. Integrating Eq. (2.7) provides $\sigma(0,t)$, which is positive and increasing.

2.2. Solution for suspension flow.
The well-posedness of the boundary/initial value problem (2.1)–(2.2) with boundary and initial data (2.4)–(2.5) was established in [1],[2], where U was taken as a constant and it was assumed that the filtration function $\lambda(\sigma)$ was C^1, $\lambda(\sigma) > 0$ for $\sigma \in [0, \phi]$. A generalization of this result follows. For $t \le \frac{\phi}{U}x$, $\sigma(x,t)$ and $c(x,t)$ vanish. For $t > \frac{\phi}{U}x$, we rewrite Eqs. (2.1)–(2.2) on characteristic lines $x - \frac{U}{\phi}t = const$ in the form

$$\frac{d\sigma}{dx} = -\lambda(\sigma)\sigma, \tag{2.8}$$

$$\frac{dc}{dx} = -\lambda(\sigma)cU. \tag{2.9}$$

where $\frac{d}{dx}$ means differentiation along characteristic lines $x - \frac{U}{\phi}t = const$ [1]. Since $c(0,t) = c_i(t)$ is specified and $\sigma(0,t)$ is obtained from solving Eq. (2.7), the family of equations (2.8)–(2.9) can be solved numerically using standard procedures for ODE's.

The following remark reflects that the direct problem is well-posed, and is useful for proving the well-posedness of the inverse problem (see appendix A).

Remark 2.1. Since the solution σ and c of the system (2.8) and (2.9) are given by an ordinary differential equations along characteristic lines, the continuity of the solution is a

consequence of the theorem on ODE solution continuity with respect to parameter changes (see [13], pag. 91). So the maps

$$\lambda \to \sigma(x, t; \lambda), \quad \lambda \to c(x, t; \lambda). \tag{2.10}$$

are continuous in the uniform norm.

Differently from [1], we assume that $\lambda(\sigma)$ is a non-negative piecewise C^1 function that may vanish in its domain $0 < \sigma < \phi$. The following Lemma, proved in [2], allows this assumption.

Lemma 2.2. *The solution of* (2.1)–(2.3) *with data* (2.4)–(2.5) *is given by* (2.8)–(2.9) *in the trapezoidal domain* $0 \leq x \leq L$, $0 < \frac{U}{\phi}t \leq x + \tau$; τ *can be infinite. If it is finite, σ is constant and equal to σ_0 in the open trapezoid* $0 < x + \tau \leq \frac{U}{\phi}t$ *and* $c(x, t) = c_i(x - \frac{U}{\phi}t)$. *Also, c is continuous in the trapezoid* $0 \leq x \leq L$, $t > \frac{\phi}{U}x$ *and σ is continuous in the infinite rectangle* $0 \leq x \leq L$, $0 < t < \infty$.

In [1] and [2] the well-posedness of the direct problem was proved. To solve inverse problems for the filtration function $\lambda(\sigma)$ and the formation damage function $k(\sigma)$, we need to solve Eqs. (2.1)–(2.5) and (2.7) many times as part of an optimization procedure. Thus, it is necessary to solve this system with high speed and accuracy.

3. Recovery methods

In this section, the function coefficients are recovered by using optimization procedures to minimize functionals that represent the difference between the solution of the direct problem and the available experimental data. Taking into account that parameter estimation in the system of partial differential equations (2.4)–(2.9) is inherently unstable ([2],[9]), i.e., the recovered parameters do not depend on the data in a stable way, we use Tikhonov's regularization, which enables us to obtain stable approximations of ill-posed inverse problems ([19]). The well-posedness of the inverse problem of recovering filtration and permeability reduction functions is proved in [2] in the operator theory framework.

First, we recover the filtration function using the method presented below or methods presented in [1] and [2]. This first inverse problem determines the filtration function from the outlet concentration, i.e., the kinetic particle capture rate is calculated from the particle concentration history. Then we recover the permeability reduction function using the method presented below or in [2]. This second inverse problem determines the formation damage function from pressure drop, i.e., the dynamic coefficient rate that specifies the hydraulic conductivity increase due to particle retention is calculated from the history of the pressure loss on the core.

3.1. **Recovering the filtration function.** In this section we formulate our method for solving the inverse problem of finding the filtration function from the effluent concentration history measured in laboratory experiments.

Before the recovery procedure, a parametrization $\lambda(\sigma; \theta)$ must be chosen for the filtration function, where θ is the set of parameters. The form of these parametric functions and their parameter ranges are dictated by physical properties of the filtration function. Then we minimize a functional relating the filtration function and the effluent concentration:

$$F^c(\theta, \alpha) = \int_B^A (c(L, t; \theta) - c_{exp}(t))^2 dt + \alpha^2 ||\theta - \theta^*||^2. \tag{3.1}$$

Here $c_{exp}(t)$ represents the effluent particle concentration history measured in the laboratory, $c(L, t; \theta)$ is obtained by solving Eqs. (2.4)–(2.9) for a fixed θ, $B = \phi L/U$ is the breakthrough time, A is the end time of the experiment and α is the regularization parameter.

3.2. Recovering the permeability reduction function.

For one-dimensional flow in a rock core, we divide Eq. (2.3) by $1/k(\sigma(x, t))U$, and integrate the resulting equation in $[0, L]$ to obtain the following relationship between deposited particle distribution and pressure drop history:

$$-\int_0^L \frac{dx}{k(\sigma(x, t))} = \frac{k_0}{\mu U}\Delta p(t), \quad 0 \le t \le A. \tag{3.2}$$

Like in the previous subsection, first we choose a parametrization $k(\sigma; \beta)$ for the permeability reduction function, with parameter set β. Then we minimize the functional:

$$F^p(\beta, \gamma) = \int_B^A \left(\Delta p(t; \beta) - \Delta p_{exp}(t)\right)^2 dt + \gamma^2 ||\theta - \theta^*||^2, \tag{3.3}$$

where $\Delta p(t; \beta)$ is the right hand side of Eq. (3.2), $\Delta p_{exp}(t)$ is the experimental data, γ is the regularization parameter.

Notice that the evaluation of $F^p(\beta, \gamma)$ requires the system of equations (2.4)–(2.9) to be solved using the filtration function recovered from the effluent concentration history: once we have the deposition of particles $\sigma(x, t)$ then the integrals in Eq. (3.3) can be evaluated.

4. NUMERICAL RESULTS WITH SYNTHETIC DATA

In this section we present two examples where the functionals (3.1) and (3.3) are minimized numerically.

We use synthetic data to calibrate the model and to test the algorithm. These data are obtained by fixing the parameters (θ, β) and solving the direct problem given by the system of equations (2.4)–(2.9) and the integral equation (3.2). We simulate observational error in real data adding random errors to the exact results. We prescribe permeability reduction and filtration functions $\lambda(\sigma) = \max\{0, 1 - 171\sigma\}$ and $k(\sigma) = (1 + 300\sigma + 100\sigma^2)^{-1}$, which are similar to those typically obtained from experimental data, and create two sets of synthetic data introducing random perturbations of the order of 0.01 ("clean data") and 0.05 ("noisy data"), shown in Figure 4.1.

The predicted effluent concentrations have relative errors of 1.8×10^{-11} and 4.7×10^{-10} for clean and noisy data, respectively, whereas the predicted pressure drop histories have relative errors of 5.6×10^{-5} and 1.4×10^{-3}. As shown in Figure 4.2, we obtained excellent matches in all cases.

In Figure 4.3 the corresponding recovered filtration and permeability reduction functions are shown. We see that for both clean and noisy data the permeability reduction and filtration functions are recovered accurately, with smaller relative errors for clean data, as expected.

In order to remedy the ill-posedness of the problem we utilize non-zero penalization parameter α and γ in Eqs. (3.1) and (3.3). To estimate these parameters, we made experiments with large values and then we reduced them until an adequate stabilization was obtained. This is a practical solution; however, a careful determination of these parameters can be done based on the noise statistics ([19]).
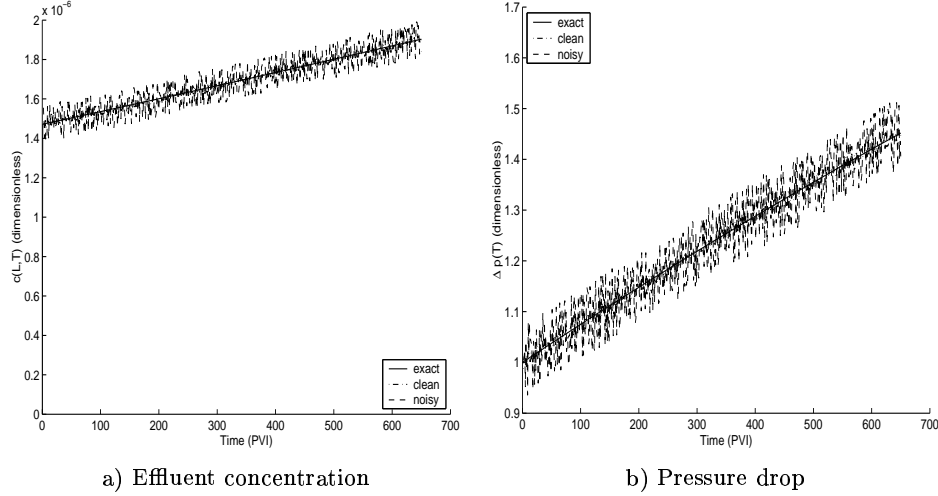
a) Effluent concentration   b) Pressure drop

FIGURE 4.1. Synthetic data.

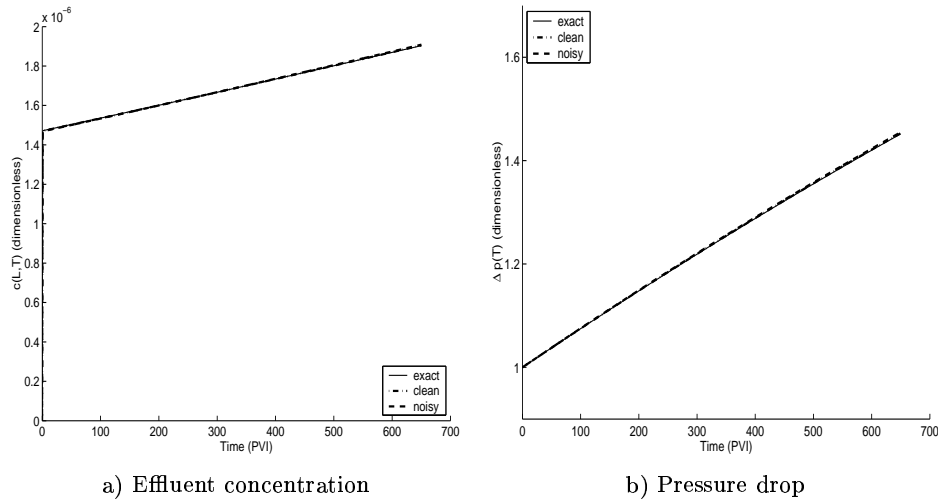a) Effluent concentration   b) Pressure drop

FIGURE 4.2. Comparison of direct problem solutions using the originally pre-
scribed parameters and those recovered from data.

Our tests show that a more stable solution is obtained in the presence of the penalization term, i.e., changes in $\triangle p_{exp}(t)$ do not produce significant changes in the recovered functions in solutions obtained with penalization. In this experiment with synthetic data, the penalization term yields no significant accuracy improvement; however, among equally inaccurate approximate solutions we prefer the stable ones, so a penalizing term is actually used in Eqs. (3.1) and (3.3). These observations are part of the sensitivity analysis, which we do not include in this paper: see [2] for a discussion on sensitivity.

The numerical examples based on synthetic data suggest that the recovery method described here is appropriate for finding the permeability reduction and filtration functions from experimental data.
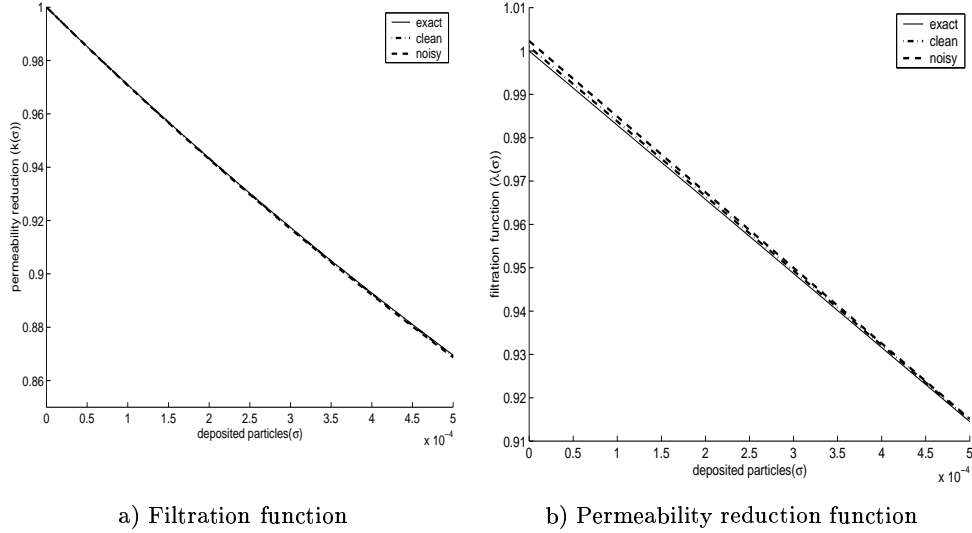
a) Filtration function
b) Permeability reduction function

FIGURE 4.3. Recovered functions for clean and noisy synthetic data.

5. NUMERICAL RESULTS FOR EXPERIMENTAL DATA

Data from Kuhnen et alii. We applied the method to the effluent particle concentration experimental data described by Kuhnen et alii in [15], shown in Figure 5.1. In these experiments, hematite suspensions of equal concentration and varying ionic strengths were injected at an equal rate into sandstone, to investigate the relation between filtration phenomena and electrostatic attraction between oppositely charged particle and porous medium surfaces. In order to apply our empirical model, we tested numerous parametrizations on each of the six data series contained therein, and obtained the following results.
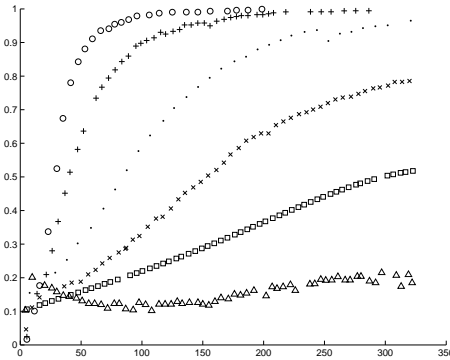

FIGURE 5.1. Data series from [15].

The data series all show an increasing trend, three of them going close to the zero filtration situation, where $c_e = c_i$ and $\lambda(\sigma) = 0$. The other three do not approach this boundary. For the model to reproduce the first case accurately, i.e. to approach $\lambda(\sigma) = 0$ as σ increases, this trend must be taken into account: all parametrizations were of the form $\lambda_0 e^{-f(\sigma)}$, where $f(\sigma)$ was a non-negative increasing function. Polinomials of various degrees yielded different results: the best all-around parameterization was $\lambda_0 e^{-\theta\sigma^2}$ for all six data series, but a good match over the $c_e \approx c_i$ segments of the first two series was only obtained using $f(\sigma) = \theta\sigma^4$.

Figure 5.2 shows good matches obtained using the quadratic exponent. Figure 5.3 shows the difference of the two best parameterizations for series approaching the limit $c_e/c_i \cong 1$.
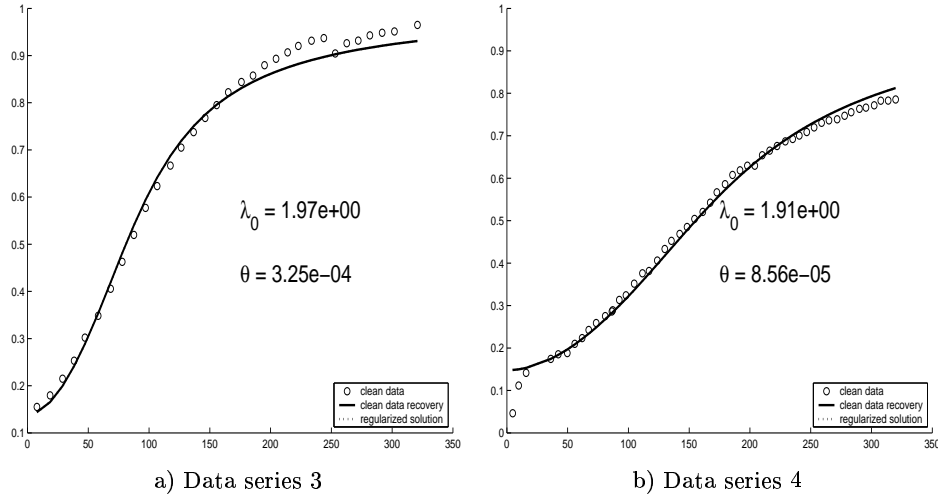

a) Data series 3   b) Data series 4

FIGURE 5.2. Best fits for effluent concentration using $\lambda(\sigma) = \lambda_0 e^{-\theta\sigma^2}$.

a) $\lambda(\sigma) = \lambda_0 e^{-\theta\sigma^2}$ b) $\lambda(\sigma) = \lambda_0 e^{-\theta\sigma^4}$
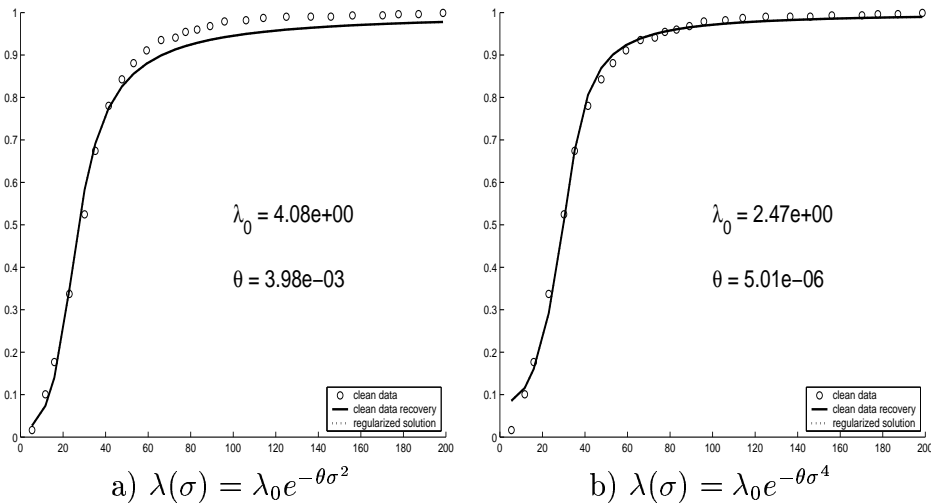
FIGURE 5.3. Profiles recovered using two fits for λ in the first data series.

Restricting the input data to the first 70 PVI of the last four data series, shown as markers in Figure 5.4a, we recovered the filtration functions shown in Figure 5.4b. We calculated the effluent concentrations using these filtrations and plotted them over the input data in Figure 5.4a, with which they coincide visually.

Data from Soma and Papadopoulos. Soma et al. performed a series of four experiments injecting oil-in-water emulsions into quartz sand [18], using similar conditions and varying the ionic strength of the emulsion, for which thei measured both effluent concentration and total permeability reduction, i.e., the pressure drop history. We applied the empirical model to the two experiments with higher ionic strength, where there was enough deposition to

a) Cropped data and model prediction.　　b) Corresponding filtration functions.
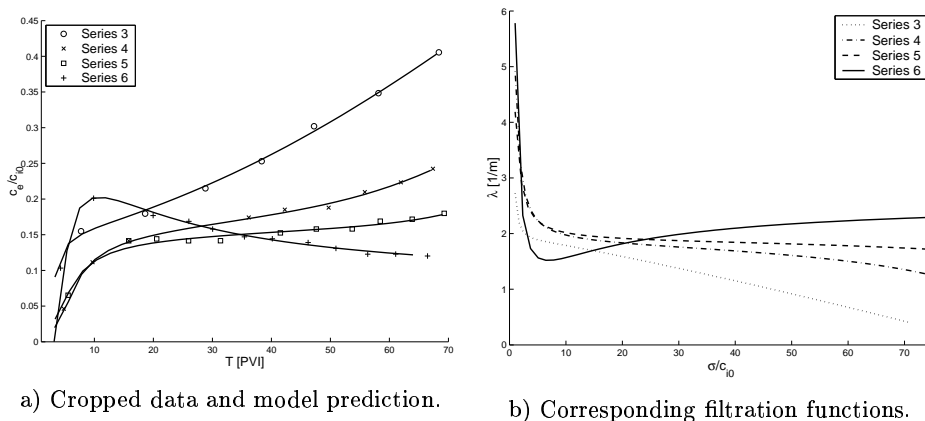
FIGURE 5.4. Excellent recovery obtained using the optimization solution shown over smooth approximations of the actual data.

prevent the effluent concentration curve from reaching $c_e/c_i \cong 1$ quickly, and to produce significant permeability reduction.

However, the effluent concentration history curves are not monotone in these two experiments. The authors attribute this lack of monotonicity to a greater attraction of oil droplets in the suspension to previously deposited oil than to the bare pore surface. In terms of the empirical $\lambda(\sigma)$ coefficient, this translates to having a non-monotone filtration function, one that increases after a certain value of σ is reached. To account for this behavior, we chose the parameterization

$$\lambda(\sigma) = \lambda_0 \bigl(e^{-\theta_1 \sigma^2} + \theta_2 \sigma\bigr) \tag{5.1}$$

For the permeability reduction function, we used the inverse of a second degree polynomial, which is compatible with the literature and suitable for our calculations. Higher degree polynomials did not yield better results.

For both data sets, the model reproduced well both the effluent concentration and permeability reduction curves, as shown in Figure 5.5. The parametric empirical functions recovered solving the inverse problems are shown in Figure 5.6.

6. CONCLUSION

The recovery method described here consists of solving two inverse problems: in the first one the filtration function is determined from the effluent concentration evolution, and in the second one the formation damage function is determined from the pressure drop history. In the cases examined, the method yields good matches between experimental and predicted data. Thus, it constitutes a viable procedure for the parameter estimation problem of determining the empirical injectivity damage functions from pressure drop and effluent concentration histories.

The method is robust and flexible, thus allowing the analysis of experimental data that other recovery methods, such as the one proposed in [1], cannot analyze. At the same time, it is easier to implement than other methods equally adequate for practical purposes such as the method for the permeability reduction developed in [2]. It is also flexible in the sense that, by accommodating different physical conditions under a minimal set of macroscopic parameters, all built into the recovered permeability damage and filtration functions, it can
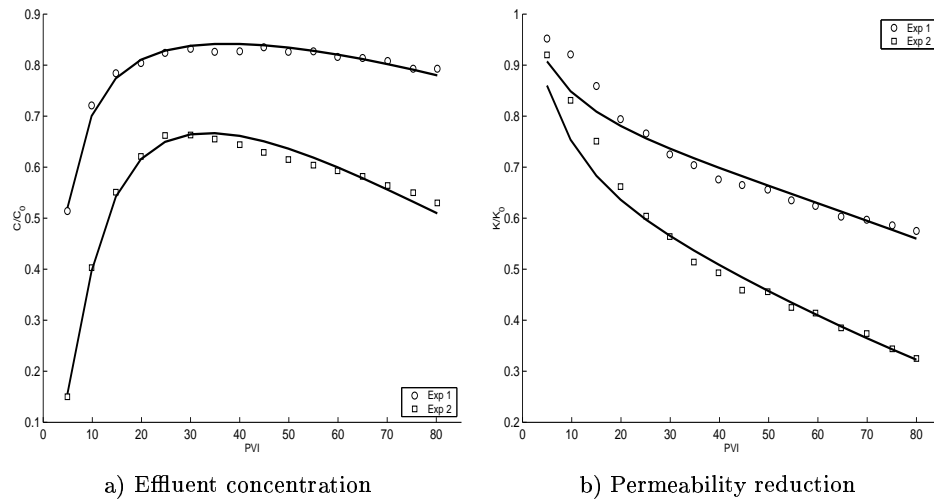
a) Effluent concentration   b) Permeability reduction

FIGURE 5.5. Recovered histories and original data in two experiments.

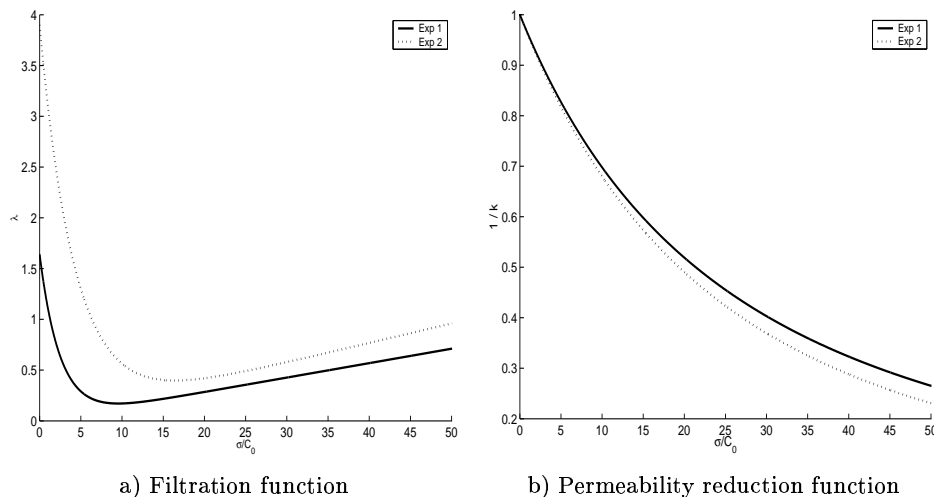a) Filtration function   b) Permeability reduction function

FIGURE 5.6. Recovered empirical functions in two experiments.

be readily applied to variations of the model (2.1)–(2.3) which cannot be solved numerically by (2.8)–(2.9).

The proposed method can be recommended for determining the filtration and formation damage functions from laboratory corefloods for use in predicting the injectivity decline, formation damage and contaminant propagation in petroleum and environmental engineering projects.

APPENDIX A. WELL-POSEDNESS OF THE FILTRATION INVERSE PROBLEM

In this section we prove that the inverse problem formulated in Eq. (3.1) is well-posed in the sense of Tikhonov ([16]). To do so, a feasible subset of parameters is chosen, such that the minimization problem has a unique minimum, and that small perturbations of experimental data produce small parameter variations. We rewrite the inverse problem in the framework of operator theory. Several results on regularization of nonlinear operators are used to prove the well-posedness of the inverse problem ([10]).

The stability and convergence results obtained here are based on [5], [10], [14] and [7]. We choose

$$D(G^c) = \{\lambda \in H^2[0,1], \quad \text{such that} \quad \lambda > \bar{\lambda}_1\}, \tag{A.1}$$

with $\bar{\lambda}_1$ constant. Let us define the nonlinear operator

$$G^c : D(G^c) \subset H^2[0,1] \to L^2[0,A], \quad G^c(\lambda) = c(1, \cdot \; ; \lambda), \tag{A.2}$$

where λ represents the filtration function and $c(1, \cdot \; ; \lambda)$ is the effluent concentration obtained from the solution of the system (2.5), (2.8)–(2.9). From the well-posedness of the direct problem ([2]), for each λ there exists a unique function $c(1, \cdot \; ; \lambda)$, so the operator in (A.2) is well-defined. Notice that the domain $D(G^c)$ is closed and convex, therefore it is weakly closed. Let us define

$$\mathcal{M} = \{\lambda \in H^2[0,1] \quad \text{such that} \quad \bar{\lambda}_2 < \lambda < \bar{\lambda}_3\}, \tag{A.3}$$

where $\bar{\lambda}_2$ and $\bar{\lambda}_3$ are constants. We have following:

Theorem A.1. *Let λ represent the filtration function and $c(1, \cdot \; ; \lambda)$ the solution of the system (2.1)–(2.2), with initial and boundary condition given in (2.4) and (2.5). Let $\eta \geq 0$ and the domain*

$$D_\eta(G^c) = \{\lambda \in H^{2+\eta}[0,1] \quad \text{such that} \quad \lambda > \bar{\lambda}_2\}. \tag{A.4}$$

The following assertions are valid.

i) The (nonlinear) operator

$$G^c : D(G^c) \subset H^{2+\eta}[0,1] \to L^2[0,A], \quad G^c(\lambda) = c(1, \cdot \; ; \lambda), \tag{A.5}$$

is continuous and injective.

ii) Let $D(G^c)$ be as in Eq. (A.1). Then the operator in (A.2) is weakly closed and compact.

iii) The map $G^c : \mathcal{M} \to G^c(\mathcal{M})$ is continuous and has continuous inverse.

Proof: (i) Let $\lambda_n \to \lambda$ in $H^{2+\eta}[0,1]$ with $\eta \geq 0$. Since $H^{2+\eta}$ is compactly embedded in $C^1[0,1]$ then $\lambda_n \to \lambda$ uniformly in $C^1[0,1]$, from Remark 2.1 it follows that $G^c(\lambda_n) \to G^c(\lambda)$ in $L^2[0,A]$. The injectivity is a consequence of the uniqueness of the solution of the system of equations (2.1)–(2.5) and (2.7).

(ii) Let $\{\lambda_n\}$ be a sequence in $D(G^c)$ converging weakly in $H^2[0,1]$ towards λ. Since $D(G^c)$ is weakly closed, then $\lambda \in D(G^c)$ and since $H^2[0,1]$ is compactly embedded in $C^1[0,1]$, then $\lambda_n \to \lambda$ in $C^1[0,1]$. By (i) $G^c(\lambda_n) \to G^c(\lambda)$ in $L^2[0,A]$. Thus, G^c is compact, hence weakly closed.

(iii) Notice that G^c is continuous in $\mathcal{M}$ by (ii). Moreover, $\mathcal{M}$ is a compact subset of $C[0,1]$, because it consists of uniformly bounded functions in $H^1[0,1]$ ([8]). Therefore (iii) is a consequence of the Lemma of Tikhonov. $\square$

From (ii) in Theorem $A.1$ and Proposition 10.1 in [10], the inverse problem of determining the filtration function λ in $G^c(\lambda) = b$ with given $b = c_e(\cdot)$ is locally an ill-posed problem.

Hence, the Regularization of Tikhonov is used to find stable solutions. The regularized solution is determined as the minimizer over $D(G^c)$ of the functional

$$\lambda \to ||G^c(\lambda) - c_e(\cdot)||^2_{L^2[0,A]} + \alpha^2||\lambda - \lambda^*||^2_{H^1[0,1]}, \tag{A.6}$$

where α is the regularization parameter. Since G^c is weakly closed, stability and convergence of the regularized solution follow from Theorem 10.2 and 10.3 in [10].

Let us denote by θ a parametrization of the filtration function $\lambda(\sigma)$. Neglecting the interaction between the parameters, i.e., by assuming that they are uncorrelated, we obtain

$$||\lambda - \lambda^*||^2_{H^1[0,1]} \approx ||\theta - \theta^*||^2 \tag{A.7}$$

where $|| \cdot ||$ denotes some appropriate norm in the parameter space. Thus, from Eq. (A.7), we see that the penalization functional can be written in terms of the parameters θ.

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