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Theoretical Definition of Formation Damage Zone With Applications to Well Stimulation

Flow of particulate suspension in porous media with particle retention and consequent permeability reduction is discussed. Using analytical model for suspension injection via single well, the permeability damage zone size was defined and expressed by transcendental equation. Analysis of field data shows that usually the size of damaged zone does not extend more than 1 m beyond the injector. The definition of damage zone size is used for design of well stimulation via deposition removal. [DOI: 10.1115/1.4001800]

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1 Introduction

Deposition of solid phase and retention of particles from the moving fluids during flow in porous media result in permeability decline [1]. In the petroleum industry, it happens in almost all processes of oil production: injection of sea or produced water with solid or liquid particles causes decline of injectivity, invasion of drilling fluid into formation rock yields decreased return permeability, and precipitation of salts in the near well region causes well index decline [2,3]. The above mentioned phenomena, called formation damage, can seriously impact on the economics of field development.

The retention and deposition phenomenon occurs also in disposal of industrial wastes, geothermal power production, and in several environmental and chemical engineering processes [4,5].

Different well stimulation technologies are used for damage removal and mitigation: acidizing, perforation, solvent, or inhibitor injection. Optimal planning of well stimulation requires knowledge of the deposit proximity relative to the wellbore [1,2]. Area of solid phase retention and deposition can be found from mathematical modeling, field data, or laboratory tests. Yet, a theoretical definition of the size of the formation damage zone, to the best of our knowledge, is not available in the literature.

In the current work, we concentrate on well index decline during injection of seawater or produced water, which is a widespread phenomenon in waterflooding field projects. Solid and oily particles are captured by rock from the injected fluid resulting in permeability decline.

The traditional model for particulate suspension in porous media consists of mass balance for suspended and retained particles, equation of particle capture kinetics, and modified Darcy's law accounting for permeability decline due to particle retention [5–10]. The problem of axisymmetric injection into clean bed allows for analytical solution [11,12]. The model contains two phenomenological parameters—filtration and formation damage coefficients. The parameters can be found from either laboratory coreflood tests or well injectivity history by solving the inverse problems [11–15].

Several micromodels for injectivity decline have been derived for pore scale: population balance equations [16,17], random walk models [18–20], and numerical network models [2,4]. The traditional model for deep bed filtration can be derived from microscale only for the case of monodispersed suspensions [17].

In the current work, we define size of formation damage zone using an analytical model for axisymmetric suspension flow in porous media. This size can be found from transcendental equation. It was shown that the formation damage size has an order of magnitude of the mean distance of deposition from the well. Calculations show that, with the exception of very low filtration coefficient cases, the damaged size almost always does not exceed 1 m.

The defined damaged zone size is proposed for application in the design of well acidizing and perforation. Some field cases presented show that in successful applications, the amount of injected acid has the same order of magnitude as that calculated by the proposed method.

The structure of this paper is as follows. The formulation of the problem for defining the formation damage zone size for the purposes of well stimulation design is presented in Sec. 2. Section 3 contains basic equations for suspension flow in porous media followed by Sec. 4 with the analytical solution for axisymmetric flow. The derivation of formation damage zone radius and the calculation results are presented in Sec. 5. Section 6 discusses applications of the damage size definition in well stimulation design.

2 Formulation of the Problem

Well efficiency is expressed by well index, which is defined as the ratio between the rate and pressure drop between the well and surrounding contour. For the case of axisymmetric flow around injection well, the well injectivity index can be determined from the steady state solution of flow equation [1,2].

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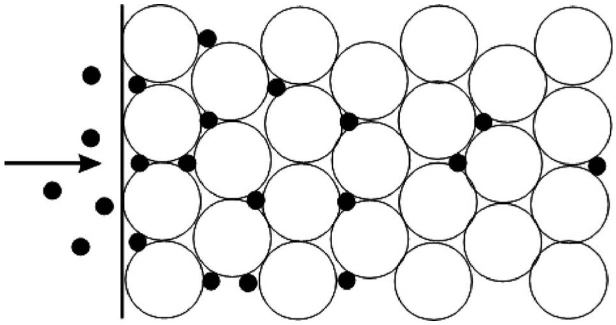

Fig. 1 Retention of particles in rock during suspension injection

$$\text{II} = \frac{q}{p_w - p_c} = \frac{2\pi k}{\mu \ln\frac{R_c}{r_w}} \tag{2.1}$$

The normalized reciprocal to injectivity index is called impedance,

$$J(t) = \frac{\text{II}(t=o)}{\text{II}(t)} = \frac{q(t=o)\Delta p}{\Delta p(t=0)}\frac{(t)}{q(t)}, \quad \Delta p = p_w - p_c \tag{2.2}$$

and is also used for formation damage description [11,15]. Decrease of injectivity index (2.1) corresponds to an increase in impedance.

Consider the injection of sea, produced, or any poor quality water into oil reservoir. The injected water contains suspended solid and liquid particles. The particles are captured in rock during the injected water flow. Figure 1 shows the particle retention in porous space. The retention is caused by different physics mechanisms: size exclusion, molecular and electric attraction, gravity segregation, and diffusion (Fig. 2).

The cross sectional area perpendicular to the fluid flow is proportional to $2\pi r$ (Fig. 3(*a*)); therefore, remote deposition results in less well damage than that near to the wellbore [11]. The retained particle concentration gradually decreases from injection sand face deep into the reservoir (Fig. 1) because the probability for particle to be captured is proportional to its trajectory length. Therefore, the effect of particle retention on well index decreases with radius. So, beyond some radius, the influence of any retained particles on well index is negligibly small. This radius is defined as the size of formation damage zone.

Consider the injection of acid or solvent in order to remove deposition. Assume that the applied chemical reacts with the particle matter and results in a complete removal of a particle.

Generally speaking, full restoration of the initial injectivity is achieved under the removal of all retained particles. Yet, deposition takes place throughout the overall swept zone. Thus, the problem is to determine penetration radius such that particle removal from this zone will lead to restoration of the bulk of injectivity.

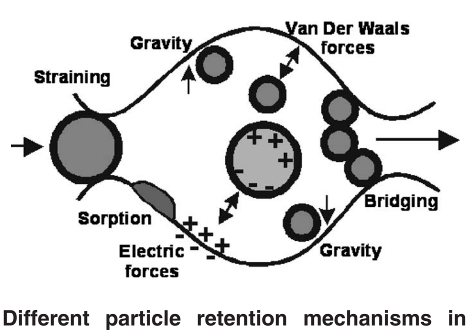

Fig. 2 Different particle retention mechanisms in porous media

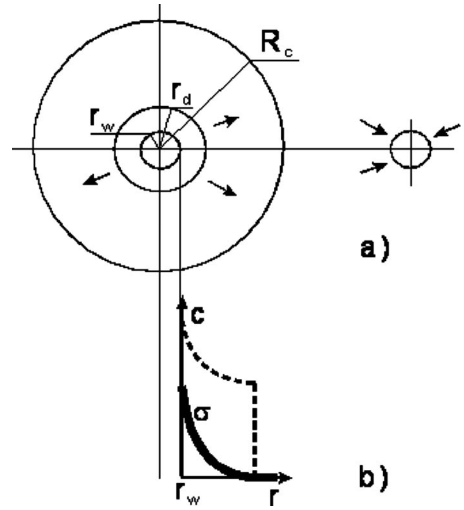

Fig. 3 Propagation of suspended and retained concentration profiles from injection to producing well. (*a*) Location of injector and producer; well, damaged, and contour radii. (*b*) Profiles of suspended and retained concentrations.

Similarly, damaged open-hole injector can be stimulated through perforation. The problem is to determine a hole length such that perforation bypasses the damaged zone and restores the initial well injectivity.

3 Mathematical Model for Axisymmetric Flow of Suspensions in Porous Media

The system of governing equations for radial transport of suspensions in porous media consists of equations of mass balance for suspended and deposited particles, kinetics of particle captured by matrix, and modified Darcy's law (momentum balance equation) [6,7].

Assuming incompressibility of carrier water and additive volumetric law of particle mixing with water yields to the following form of continuity equation for axisymmetric flow

$$\phi\frac{\partial c}{\partial t} + \frac{q}{2\pi r}\frac{\partial c}{\partial r} = -\frac{\partial \sigma}{\partial t} \tag{3.1}$$

where c and σ are the concentrations of suspended and retained particles, respectively, ϕ is the porosity, and q is the volumetric water flow rate.

Particle capture rate is proportional to advective particle flux cU with coefficient of proportionality λ',

$$\frac{\partial \sigma}{\partial t} = \lambda' U c \tag{3.2}$$

where $U=q/2\pi r$ is the linear velocity of carrier water and λ' is the filtration coefficient. The filtration coefficient is equal to the probability of a particle being captured by the matrix per unit of the particle trajectory [7,17]. Since the only characteristic of the capture mechanism, used in the analysis, is the capture probability, formula (3.2) is valid for any combination of capture mechanisms.

Modified Darcy's law accounts for permeability decrease due to retained particles and residual oil near the injection well,

$$U = -\frac{k_0 k_{\text{rwor}}}{(1+\beta\sigma)\mu}\frac{\partial p}{\partial r} \tag{3.3}$$

where μ is the viscosity of injected aqueous suspension, p is the pressure, and k_0 is the initial permeability of retained-particle-free porous media. Permeability decrease due to retained particles depends on the captured particle concentration and on the formation damage coefficient β. Decrease of absolute permeability for water under the presence of residual oil yields the multiplier k_{rwor}.

Let us introduce dimensionless radial coordinate, time, suspended and captured concentrations, pressure, and filtration coefficient:

$$\rho = \frac{r}{R_c}, \quad X = \rho^2, \quad T = \frac{1}{\phi \pi R_c}\int_0^t q(\tau)d\tau, \quad C = \frac{c}{c_0},$$
$$S = \frac{\sigma}{\phi c_0}, \quad \lambda = \lambda' R_c \tag{3.4}$$

where c^0 is the concentration of particles in the injected suspension, and R_c is the contour radius, which is equal to the half-distance between injection and production wells (Fig. 3).

Through substitution of dimensionless variables (3.4) into the system, the governing equations (3.1)–(3.3) for dimensionless coordinates and parameters take the forms

$$\frac{\partial C}{\partial T} + \frac{\partial C}{\partial X} = -\frac{\partial S}{\partial T}$$
$$\frac{\partial S}{\partial T} = \frac{\lambda C}{2\sqrt{X}} \tag{3.5}$$
$$\frac{1}{X} = -\frac{2}{(1+\beta\phi c^0 S)}\frac{\partial P}{\partial X}$$

The assumption that the capture rate is independent of pressure yields to the separation of the first and second equations from the third equation, i.e., the system of two equations with unknown suspended and retained concentrations can be solved separately from the equation for pressure $P(X,T)$.

Injection of water with constant particle concentration c_0 into a "clean" bed results in the following initial and boundary conditions:

$$T = 0{:}C = S = 0, \quad X = X_w{:}C = 1 \tag{3.6}$$

Here the constant concentration boundary condition is set on the sand face of the injection well, where X_w corresponds to well radius r_w.

4 Analytical Solution for Injection Into a Single Well

The system of first and second equations of Eq. (3.5) subject to initial and boundary conditions (3.6) can be solved using method of characteristics, allowing for explicit expressions for suspended and retained concentrations [11,12,15],

$$C(X,T) = \begin{cases} e^{-\lambda(\sqrt{X}-\sqrt{X_w})}, & X < X_w + T \\ 0, & X > X_w + T \end{cases} \tag{4.1}$$

$$S(X,T) = \begin{cases} \dfrac{\lambda e^{-\lambda(\sqrt{X}-\sqrt{X_w})}}{2\sqrt{X}}(T - X + X_w), & X < X_w + T \\ 0, & X > X_w + T \end{cases} \tag{4.2}$$

Figure 3(*b*) shows the profiles of both concentrations for a fixed time. The suspended and retained concentrations equal zero ahead of the injected water front $X=X_w+T$. The suspended concentration decreases from c_0 at the injection well down to some positive value on the water front. The retained concentration decreases from some positive value at the injector down to zero at the injected water front.

Figure 4 shows the trajectory of the front $X=X_w+T$. Coordinates $X=X_w$ and $X=1$ corresponds to injection well and contour, respectively.

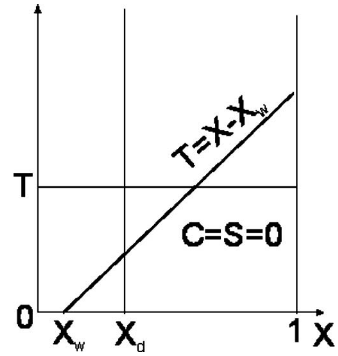

Fig. 4 Propagation of concentration front in plane of dimensionless distance and time (X,T)

Let us calculate the pressure drop between the injection well and the contour. Expressing pressure gradient from Eq. (3.5) results in

$$\Delta P = \int_{X_w}^{1}\left(-\frac{\partial P}{\partial X}\right)dX = \int_{X_w}^{1}\frac{1+\beta\phi c^0 S(X,T)}{2X}dX = -\frac{1}{2}\ln X_w + \frac{\beta\phi c^0}{2}\int_{X_w}^{1}\frac{S(X,T)}{X}dX \tag{4.3}$$

Substitution of formula for retained particle concentration into the previous expression yields

$$\Delta P = -\frac{1}{2}\ln X_w + \frac{\beta\phi c^0}{2}\int_{X_w}^{X_w+T}\frac{\lambda e^{-\lambda(\sqrt{X}-\sqrt{X_w})}}{2X\sqrt{X}}(T - X + X_w)dX \tag{4.4}$$

Let us substitute a new variable $y=\lambda\sqrt{X}$ into the integral in

$$\int_{X_w}^{X_w+T}\frac{S(X,T)}{X}dX = T\lambda^2\left(e^{y_w}ei(y_w) - e^{y_w}ei(\lambda\sqrt{T}) - e^{y_w}\frac{e^{-\lambda\sqrt{T}}}{\lambda\sqrt{T}} + e^{y_w}\frac{e^{-y_w}}{y_w}\right) + e^{y_w}e^{-\lambda\sqrt{T}} - 1 \tag{4.5}$$

Expression (4.5) contains six terms. Let us evaluate them.

We consider volumes of injected water that highly exceed the well volume, so

$$y_w \ll \lambda\sqrt{T}, \quad ei(y_w) \gg ei(\lambda\sqrt{T}) \tag{4.6}$$

allowing the second term in brackets of the right hand side of Eq. (4.5) to be neglected when compared with the first term. For the same reason, the following inequality holds:

$$\frac{e^{-\lambda\sqrt{T}}}{\lambda\sqrt{T}} \ll \frac{e^{-y_w}}{y_w} \tag{4.7}$$

So, the third term in brackets of Eq. (4.5) can be neglected if compared with the fourth term.

The fourth term is equal to $T\lambda^2/y_w$. For the typical values r_w $=0.1$ m, $R_c=100$ m, $T=1$, and $\lambda'=1$ 1/m it is equal to 107, allowing the sixth term in Eq. (4.5) (unity) to be neglected when compared with the fourth term.

Using the same values, $y_w-\lambda\sqrt{T}=-100$ leading to the following inequality:

$$e^{y_w}e^{-\lambda\sqrt{T}} \ll 1 \tag{4.8}$$

The fifth term in Eq. (4.5) can be neglected if compared with unity.

Finally, integral in Eq. (4.5) takes the form

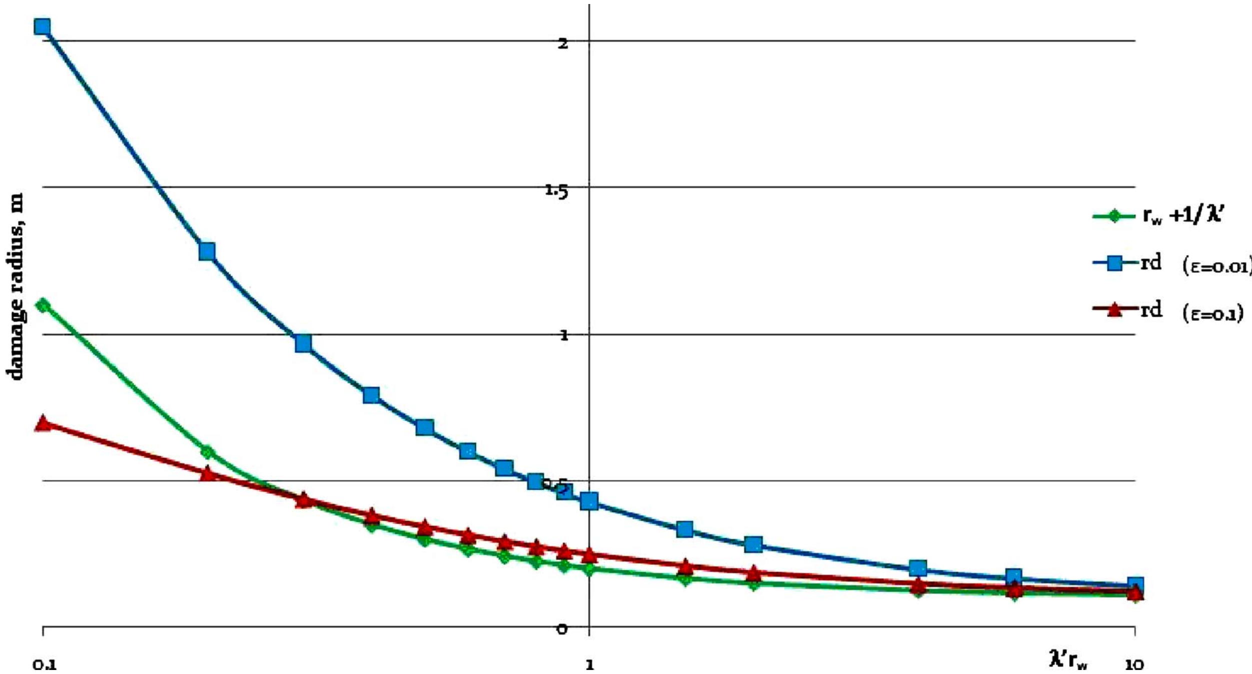

Fig. 5 Comparison between formation damage zone size and mean distance to particle deposit

$$\int_{X_w}^{X_w+T}\frac{S(X,T)}{X}dX = T\lambda^2 e^{y_w}ei(y_w)+\frac{T\lambda^2}{y_w} \tag{4.9}$$

resulting in the final expression for pressure drop between the injector and the contour

$$\Delta P = -\frac{1}{2}\ln X_w + \frac{T\lambda^2\beta\phi c^0}{2}\left(e^{\lambda' r_w}ei(\lambda' r_w)+\frac{1}{\lambda' r_w}\right) \tag{4.10}$$

The first term in Eq. (4.10) corresponds to the case where deposition is yet to take place ($T=0$). The second term is entirely responsible for permeability damage and is called the skin factor [1,2],

$$\Delta P = -\tfrac{1}{2}\ln(X_w)+Sk \tag{4.11}$$

From Eq. (4.3) accounting for Eqs. (4.4) and (4.10) follows a formula for the skin factor,

$$Sk(T)=\frac{\beta\phi c^0}{2}\int_{X_w}^{X_w+T}\frac{S(X,T)}{X}dX = \frac{T\lambda^2\beta\phi c^0}{2}\left(e^{\lambda' r_w}ei(\lambda' r_w)+\frac{1}{\lambda' r_w}\right) \tag{4.12}$$

5 The Damaged Zone Radius

In this section, we define size of formation damage zone with injection of suspended particles and compare it with the mean distance to the deposited particles.

5.1 Definition of the Damaged Zone Radius. Let us define the formation damage zone with a radius r_d such that if all particles from the r_d—neighborhood of the well—are removed, the skin factor will almost vanish. From Eq. (4.12) follows the definition of damaged zone size,

$$\frac{\int_{X_w}^{X_w+T}\frac{S(X,T)}{X}dX - \int_{X_w}^{X_d}\frac{S(X,T)}{X}dX}{\int_{X_w}^{T}\frac{S(X,T)}{X}dX} \le \varepsilon \tag{5.1}$$

where ε is a small number representing the proportion of skin to remain. The number ε is a small parameter showing the accuracy of the assumption, that particle retention outside the damaged zone does not cause the injectivity impairment.

From Eq. (5.1) it follows that the skin factor, caused by particles retained outside the neighborhood of r_d, is negligibly small.

Formula (5.1) can be simplified

$$\frac{\int_{X_d}^{X_w+T}\frac{S(X,T)}{X}dX}{\int_{X_w}^{T}\frac{S(X,T)}{X}dX} \le \varepsilon \tag{5.2}$$

Calculation of the integral in numerator of Eq. (5.2) repeats derivations (4.3)–(4.5),

$$\int_{X_d}^{X_w+T}\frac{S(X,T)}{X}dX = T\lambda^2 e^{\lambda' r_w}\left(ei(\lambda' r_d)-ei(\lambda' R_c\sqrt{T})-\frac{e^{-\lambda' R_c\sqrt{T}}}{\lambda' R_c\sqrt{T}} + \frac{e^{-\lambda' r_d}}{\lambda' r_d}\right)+e^{\lambda' r_w}e^{-\lambda' R_c\sqrt{T}}-e^{\lambda' r_w}e^{-\lambda' r_d} \tag{5.3}$$

Estimates (4.6)–(4.8) remain the same, i.e., first and fourth terms in Eq. (4.5) remain significantly larger than the other four terms. Finally, inequality (5.2) takes the form

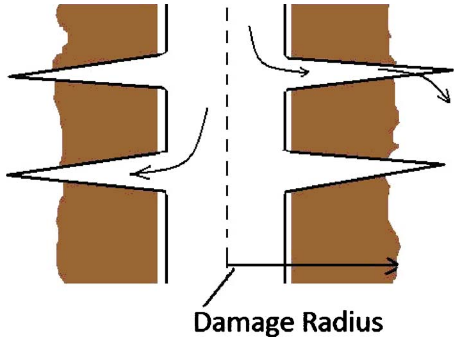

Fig. 6 Schema of perforation of damaged open-hole well and estimation of the hole length

$$\frac{ei(\lambda' r_d) + \dfrac{e^{-\lambda' r_d}}{\lambda' r_d}}{ei(\lambda' r_w) + \dfrac{e^{-\lambda' r_w}}{\lambda' r_w}} \leq \varepsilon \tag{5.4}$$

For a given ε, the formation damage zone size r_d can be determined from equality (5.4) by numerically solving the transcendental equation.

5.2 Estimates of Damaged Zone Radius. The results of calculation of damaged zone size have been performed for $\varepsilon=0.1$ and $\varepsilon=0.01$. Figure 5 presents plots of r_d versus dimensionless filtration coefficient λ for both cases. Here it was assumed that the well radius $r_w=0.1$ m. Abscissa axis has a logarithmic scale.

The plots show that values of the damaged zone size for $\varepsilon=0.1$ and $\varepsilon=0.01$ almost coincide for large values of filtration coefficient, $\lambda'>10$ 1/m. The values of r_d vary significantly for low values of filtration coefficient, $\lambda'<1$ 1/m.

The reference value of mean particle penetration during deep bed filtration is $1/\lambda'$ [7,11]. The curve $r=r_w+1/\lambda'$ is labeled in Fig. 5 by r_d^σ. The curve almost coincides with r_d-curve for $\varepsilon=0.1$ for large values of filtration coefficient, $\lambda'>1.6$ 1/m.

The case of $\lambda'>10$ 1/m covers almost all cases of laboratory coreflood tests [21]. For these cases, formula $r_d=1/\lambda'$ can be used to estimate the damaged zone radius.

The case of $\lambda'>1.6$ 1/m covers almost all field cases of well injectivity [11,15]. For these cases, formula $r_d=1/\lambda'$ can be used to estimate the damaged zone radius for $\varepsilon=0.1$. If for some reasons 99% of initial injectivity must be restored by well stimulation, equality (5.4) must be applied for $\varepsilon=0.01$.

Yet, a few cases of low filtration coefficient, $\lambda'<1$ 1/m, have been reported for high-rate injection wells [11,13]. In these cases, equality (5.4) must be applied for $\varepsilon=0.1$ and $\varepsilon=0.01$.

6 Applications of Damaged Zone Size Definition

The proposed definition of formation damage zone radius can be used for estimates of perforation length and in design of well acidification.

6.1 Sizing of Perforation Holes. Consider an open-hole injector damaged by sea or produced water injection. One of the well stimulation options is perforation of the well (Fig. 6).

The perforation hole must bypass the damaged zone in order to restore the initial value of well injectivity index or even to increase it. Therefore, the perforation hole depth must exceed the damaged zone radius r_d.

This estimate can be also useful in situation where the injectors with open-hole completion were used at the beginning of waterflooding, and subsequent injectors were perforated. The filtration coefficient can be determined from injectivity history of other open-hole wells and applied for design of perforated wells in the same reservoir.

Bedrikovetsky et al. [15] presented the filtration coefficient value as calculated from the injectivity decline history $\lambda'=10$ 1/m. The corresponding perforation length is equal to $1/\lambda'=0.1$ m. For another value $\lambda'=1.6$ 1/m reported in Ref. [13], the perforation length is $1/\lambda'=0.6$ m.

6.2 Estimate of Acid Volume for Well Stimulation. The costs associated with the acidification treatment are very high for long horizontal wells due to both the volume of acid required and long shut-in time during acid injection [2]. Thus, it is important to accurately determine the necessary volume of injected acid in order to remove the damage.

The optimal amount of injected acid depends on reactivity between the acid and retained particles and between the acid and the reservoir rock, by the cost of well shut-down period, by economic evaluation of acid treatment efficiency, etc. So, different criteria for acid treatment design do exist. To the best of our knowledge, the mathematical model accounting for the above mentioned factors and allowing optimization of the process by economic criterion is not available from the literature.

Yet the proposed criterion of damaged zone radius for estimation of the necessary acid volume may give a correct order of magnitude for an optimal acidification. In the case where the acid composition was already selected, it may be used for acid volume determination. Therefore, in order to validate the proposed criterion, below we compare the damaged zone size with acid sweep radius for the case of successful acid treatments.

Consider the injection of acid volume V,

$$V = \phi \pi r_d^2 \tag{6.1}$$

per unit of well length, where r_d is determined by skin removal criterion (5.4). Using $\varepsilon=0.1$ in Eq. (5.4) results in the removal of 90% of the skin factor, and using $\varepsilon=0.01$ causes the removal of 99% of the damage. So, the proposed criterion for acid volume (6.1) assumes the damaged zone sweep by acid.

Let us validate the proposed criterion by comparison with the field cases. Figure 7 presents the impedance growth during water injection into giant Brazilian high permeability deep-water sandstone reservoir (Campos basin) and the impedance fall after the acidizing. Impedance growth corresponds to injectivity index decline (see Eq. (2.2)). In this field, well injectivity decreased 10–15 times during 15 years of waterflooding [22]. As is presented in Fig. 7, acidizing in the late 1998 completely restored the initial injectivity index.

The amount of injected acid was 150 gal/ft, which corresponds to a penetration radius of 2.5 m for porosity $\phi=0.32$. The filtration coefficient as calculated from impedance growth curve is 0.6 1/m. The low value of filtration coefficient is explained by high rock permeability and high injection rates in this field. The value $\lambda=0.6$ 1/m corresponds to $r_d=0.7$ m and $r_d=2.6$ m for $\varepsilon=0.1$ and 0.01, respectively.

So, the successful acidizing took place under sweeping of damaged zone by injected acid, which validates the proposed criterion.

Figure 8 shows the injectivity index decline at the well from the waterflooded shallow sandstone field (Brazil, Campos basin); it shows that the injectivity index increases after the acidizing. The applied acidizing was considered to be successful—injectivity after the second well treatment was even higher than that after the first treatment.

The filtration coefficient was calculated from the injectivity decline curve after first acidizing: the filtration coefficient $\lambda=16.5$ 1/m. The damage radii are $r_d=0.3$ m and $r_d=0.4$ m for $\varepsilon=0.1$ and 0.01, respectively. The amount of injected acid was 170 gal/ft, which corresponds to a penetration radius of 2.4 m; i.e., the acid penetration depth exceeds the radius r_d of the damaged zone. Again, the sweep area radius exceeded the damaged zone size, which validates the criterion (6.1).

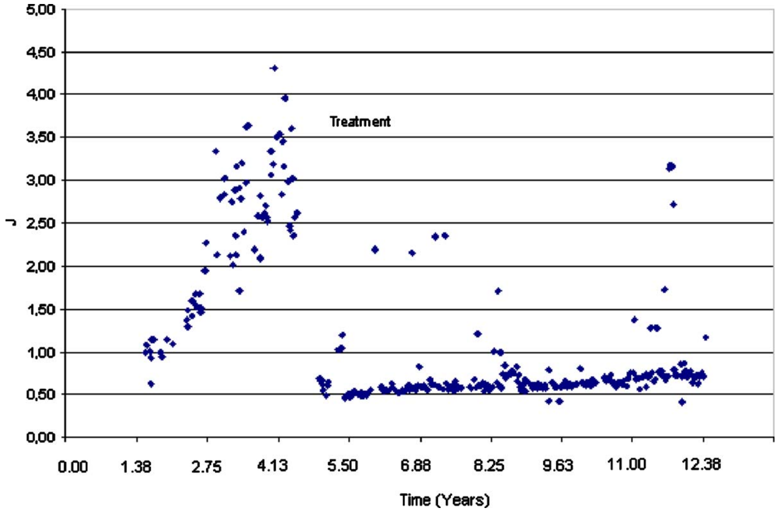

Fig. 7 Impedance increase during waterflooding and full restoration of initial injectivity after acidizing

7 Summary and Discussion

Analytical model for suspension transport in porous media for axisymmetric flow allows defining the radius of formation damage zone. An implicit formula for damaged zone size has been derived. The damaged zone radius is a function of filtration coefficient. Since the filtration coefficient can be calculated from the well injectivity history, the damaged radius can be also estimated.

The definition of the radius of formation damage zone can be used for estimation of perforation length for damaged open-hole injection wells.

Using the proposed criteria to estimate the acid volume is in agreement with several field cases where the successful treatment used higher acid volumes than that calculated from damaged zone radius.

The proposed method for determining the formation damage zone size with criterion for well stimulation design can be applied for other formation damage areas where either analytical or numerical models for permeability reduction process have been developed: sulfate scaling in production wells, carbonate scaling in injection wells, fine migration during either production of heavy

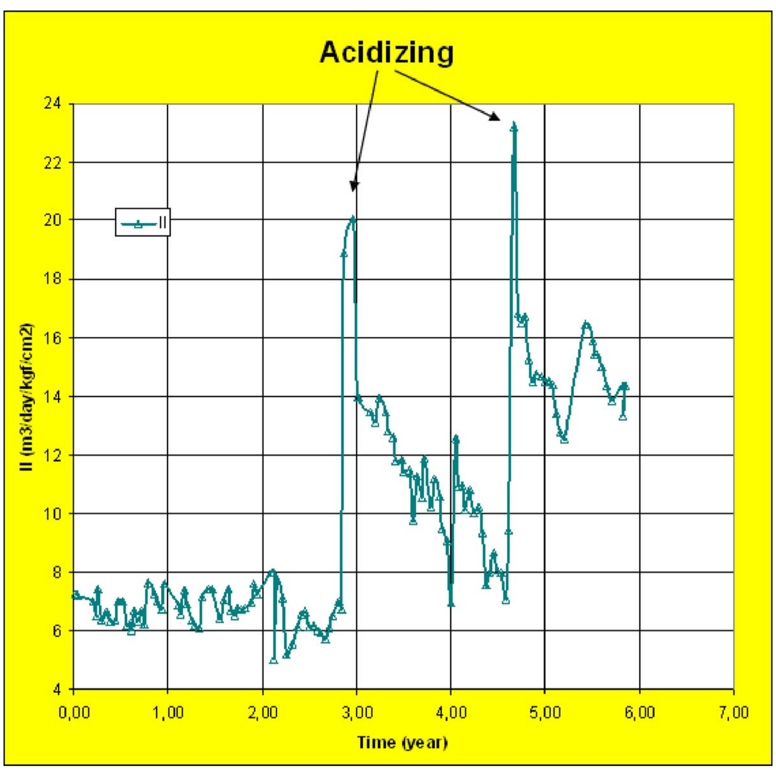

Fig. 8 The case of successful damaged well acidizing

oils or exploitation of poorly consolidated reservoirs, drilling mud invasion into oil bearing formations, water disposal into aquifers, potable water production from artesian wells, and damage removal during geothermal energy production [1–5,23].

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