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ABSTRACT

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Louise Allsopp

There are two main schools of thought which seek to explain currency crises; fundamentalist theories (Krugman, 1979) and speculative theories (Obstfeld, 1986). However, neither of these approaches considers the duration of a currency crisis. I explain the duration of a crisis in terms of a war of attrition model. Having set up the basic framework, I consider the impact of a change in the parameter values on a government's optimal time of concession. I then introduce asymmetric post stabilisation utilities into the framework.

Keywords: war of attrition, currency crisis

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1 Introduction

In January 1992, the ERM celebrated sixty months without a realignment. However, by the September, two of the currencies had the left the system (the Pound and the Lira) and the Peseta and Escudo had devalued voluntarily. There has been a vast literature in this area which debates on the causes of the collapse. The arguments put forward are summarised in the paper by Eichengreen and Wyplosz (1993). However, the literature does not explain the duration of a crisis.

The duration of a currency crisis and the subsequent collapse of the currency may be explained using a ‘war of attrition’ model such as that described by Alesina and Drazen (1991) in the context of a fiscal stabilisation.

The paper is set out as follows. Firstly, I will set out the ‘war of attrition’ model and solve it to show the optimal time of concession. I will then analyse the effect of changing the parameter values. In particular, I am concerned with the effect on the delay until one side concedes. In an extension to the model, I will introduce asymmetric post stabilisation utilities into the framework. I will then draw conclusions.

2 A ‘War of Attrition’ Model to Explain the Duration of a Crisis

The logic of the Alesina and Drazen argument is as follows. If a stabilisation has particular implications i.e. a burden to be borne by the parties in question, then each group will attempt to shift the burden onto the other. This
leads to a ‘war of attrition’ in which each group attempts to hold out in the hope that the other will concede first and bear the larger share of the burden. In the Alesina and Drazen case, the initial shock reduces available tax revenues to pay off a budget deficit and each party tries to shift the resulting tax incidence onto the other.

The framework also lends itself to the issue of foreign aid. This has been shown by Casella and Eichengreen (1996) who investigate the importance of the timing of aid. They find that foreign aid decided upon and transferred earlier in the game can lead to an early stabilisation. However, if it is decided upon and transferred late in the game, the effect can be destabilising. This encourages the further postponement of reforms.

My model uses the Alesina and Drazen approach and applies it to a currency crisis. In this model, I consider two governments; Germany and the UK. A speculative attack is launched on the Pound at time $t = 0$. This has the effect of imposing a cost on each of the member governments. This can take the form of a political burden since an attack on a currency can jeopardise the future of the ERM and thus have serious consequences for all members. It may also be thought of as purely financial in terms of undesired movements in foreign currency reserves. The attack implies that investors sell their holdings of sterling and purchase Deutschmarks. In the absence of intervention, this would generate a decrease in the UK money supply and a corresponding increase in the German money supply. However, as a temporary measure, the UK and German authorities can overcome these movements in foreign currency reserves by the use of sterilised intervention. This instrument is costly for each government and hence it becomes increas-
ingly difficult to maintain this situation. As a consequence, the governments endure a mounting pre stabilisation utility loss which is a function of the cost imposed by sterilisation.

The situation can only be restored by a fundamental change in policy undertaken by one of the governments. This could take the form of a change to fiscal policy, a change in interest rates or the abandonment of the fixed rate regime. This would halt an attack on the currency and hence resolve the currency crisis. Thus, in this context, a stabilisation is brought about by a change in policy by one of the two governments. The government carrying out the change in policy is deemed the ‘losing’ country. It therefore bears a utility loss in excess of the ‘winning’ country but this loss is smaller than that which each country was enduring prior to stabilisation.

The crucial feature of this model is that the governments differ in terms of their welfare loss and that neither government knows the welfare loss of its opponent. Hence, as time passes prior to stabilisation, each government can only make deductions about the relative strength of its opponent. Given this scenario, it is possible to calculate the optimal time of concession for a government and hence the timing of the collapse of an exchange rate regime.

3 Model

Within this framework, I consider two governments, namely the UK and Germany. However, the model can be extended to include more than two players. These governments differ in the welfare loss they suffer as a result of an attack. This is private information i.e. each government knows its
own welfare loss but does not know that of its opponents. It is assumed that reserves are undepleted before time $t = 0$. At this time, a speculative attack is launched on the Pound. This generates a cost of $c$ per period. I am assuming that this level is constant.\footnote{In the Alesina and Drazen framework, government spending before a stabilisation is financed through a combination of new bond issues and distortionary taxation. It follows that although initial expenditure is constant, there is an increase in interest payments over time caused by the rising stock of bonds. Casella and Eichengreen note that this is misleading since it implies that an increasing burden hastens the stabilisation. This is not, in fact, the case. Therefore, I present the model in a simple form with a constant cost which causes a welfare loss.}

The utility loss for each government is proportional to the size of the cost but differs across governments. Each government’s utility loss is determined by the parameter, $\theta_i$ which lies between the values, $\underline{\theta}$ and $\overline{\theta}$. It is assumed that prior to a stabilisation, the flow utilities for each government are given by:

\[
  u^D_i = - \left( \theta_i + \frac{1}{2} \right) c
\]

where $i = 1, 2$ denotes each of the two governments. Each government estimates the opponent’s cost using the cumulative probability distribution function, $F(\theta)$ and the associated density function, $f(\theta)$. For simplicity, the distribution of $\theta$ is assumed to be uniform between $\underline{\theta}$ and $\overline{\theta}$.

A resolution of the crisis implies that there is no longer a political and financial cost imposed on the governments arising from the crisis. Thus, there is an incentive to concede and end the crisis. However, the resolution of the crisis involves costly policy changes which are divided unequally between the governments. The loser bears the larger portion of the burden, $\alpha$, which is
assumed to be greater than $\frac{1}{2}$. The winner bears the smaller share of $(1 - \alpha)$. It is assumed that this share of the burden is not bargained over. Furthermore, the governments bear this cost forever. Notably, a value of $\alpha$ close to $\frac{1}{2}$ would indicate what Alesina and Drazen refer to as ‘political cohesion’ i.e. it would indicate a willingness for each country to bear approximately the same portion of the burden. I will develop this idea in the next section.

It follows that after a stabilisation, the utility losses borne by each government will be determined by the value of $\alpha$ and the cost, $c$, so that flow utility will become:

$$
u^L = -\alpha c$$

(2)

for the loser and:

$$
u^W = -(1 - \alpha)c$$

(3)

for the winner. The important point to note is that the flow utilities for the winner and loser are higher than the pre stabilisation utility. Before one side concedes, each government has a utility given by (1). The loss is determined by the cost per period and each government’s value of $\theta$. Following a stabilisation, the governments share the cost, $c$, with the winner bearing a smaller burden. It follows that it would be better to be the losing government than to endure (1). However, the crucial point is that each government does not know the strength of its opponent as given by the opponent’s $\theta$. This implies that it is optimal to wait in the hope that the opponent will concede first. The discounted lifetime utilities at the point of stabilisation are as follows:
\[ V^L = -\frac{\alpha c}{\delta} \] (4)

for the loser and:

\[ V^W = -\frac{(1 - \alpha)c}{\delta} \] (5)

for the winner. Note that \( \delta \) is the discount rate. It follows that the lifetime utility from the date at which the crisis begins of the winner and loser may be written as:

\[ U^j(T) = \int_0^T u^D(x) e^{-rx} dx + e^{-rT} V^j(T) \] (6)

where \( j = W, L \).

It is now possible to evaluate the expected utility as of time 0 as a function of the chosen concession time of a government, \( T_i \). This is the sum of \( U^W(X) \) multiplied by the probability of the opponent conceding at any time, \( X \leq T_i \) plus \( U^L(T_i) \) multiplied by the probability that the opponent has not conceded before \( T_i \). The solution of the game is the function, \( T(\theta_i) \) which maps the cost parameter, \( \theta_i \), onto its optimal time of concession, \( T_i \). The expected utility of a government can now be written as:

\[ EU(T_i) = [1 - H(T_i)] U^L(T_i) + \int_0^{T_i} U^W(x) h(x) dx \] (7)

where \( H(T) \) is the distribution of the opponent’s optimal time of concession and \( h(T) \) is the density function. Substituting (6) into (7) gives the following:
\[ EU(T_i) = [1 - H(T_i)] \left[ \int_0^{T_i} u^P(x) e^{-rx} dx + e^{-rT_i} V^L(T_i) \right] \\
+ \int_{x=0}^{x=T_i} \left[ \int_0^x (z) e^{-rz} dz + e^{-rx} V^W(x) \right] h(x) dx \] (8)

It is possible to find the optimal time of concession by finding the value of \( T_i \) which maximises (8). Differentiating (8) with respect to \( T_i \) and setting the resulting expression equal to zero gives:

\[ \frac{dEU}{dT_i} = h(T_i) \left[ V^W(T_i) - V^L(T_i) \right] \\
+ [1 - H(T_i)] \left[ u^P_i(T_i) - u^L_i(T_i) + \frac{dV^L_i(T_i)}{dT_i} \right] = 0 \] (9)

Substituting in the values of (1), (2), (4) and (5) gives:

\[ \frac{dEU}{dT_i} = h(T_i) (2\alpha - 1) \frac{c}{\delta} + [1 - H(T_i)] \left[ c \left( \alpha - \frac{1}{2} - \theta_i \right) \right] = 0 \] (10)

Differentiating with respect to \( \theta_i \) gives:

\[ \frac{d^2 EU}{dT_i d\theta_i} = - \left[ 1 - H(T_i) \right] c < 0 \] (11)

Hence the optimal concession time, \( T_i \), is monotonically decreasing in \( \theta_i \). This result is significant since it defines the relationship between \( H(T) \) which is unknown and \( F(\theta) \) which is known. This relationship is:

\[ 1 - H(T(\theta)) = F(\theta) \] (12)
Differentiating this gives:

\[-b \frac{\partial T(\theta)}{\partial \theta} \frac{\partial T(\theta)}{\partial \gamma} = f(\theta)\] (13)

The Nash equilibrium is described by the function, \(T(\theta_i)\). This defines the optimal point of concession given that the opponent is following the same decision rule. Using (10), (12) and (13), the symmetric Nash equilibrium can be described as follows:

\[T(\theta) = \frac{f(\theta)}{F(\theta)} \frac{2\alpha - 1}{\delta (\theta + \frac{1}{2} - \alpha)}\] (14)

It is assumed that a government with the highest possible cost of waiting will concede immediately. Hence this gives the boundary condition of:

\[T(\theta) = 0\] (15)

The differential equation, (14), can now be solved to find the function, \(T(\theta)\). This is given by:

\[T(\theta) = \frac{2\alpha - 1}{\delta (\theta + \frac{1}{2} - \alpha)} \left( \ln \frac{\theta + \frac{1}{2} - \alpha}{\theta - \theta} - \ln \frac{\theta - \theta}{\theta - \theta} \right)\] (16)

An additional assumption is imposed here. It is assumed that \(\theta > \alpha - \frac{1}{2}\). This implies that a government will concede in finite time. If the government possessed a \(\theta\) such that \(\theta + \frac{1}{2} < \alpha\), it would never be optimal for the government to concede because before stabilisation, it is bearing a utility loss which is smaller than the utility loss of a loser.

In summary, the working of the game is as follows. At the outset there
is a speculative attack the resolution of which imposes a large political and financial cost which is divided unequally between the governments. The winner takes on the smaller share while the loser adopts the larger part of the burden. They know what payoffs they will receive if they win or lose. At time 0 immediately following the speculative attack, there will be a probability that the opponent will concede i.e. a probability that the opponent has a $\theta = \theta^*$. If it does not concede straight away, the government realises that its opponent is not of the ‘weakest’ type. As time progresses, if the opponent still does not concede, the government learns more about it. It learns that the opponent does not have a value of $\theta$ above a particular level. This process continues until the conditional probability of the opponent conceding is such that (16) holds. This denotes the optimal time for the government to give in and accept being a loser.

I argue that the speculative attacks on the Pound in 1992 may have generated a ‘war of attrition’ set up similar to the one described above. Each government was reluctant to accept the policy changes required to halt the crisis. The UK hoped for outside support while Germany had financial commitments elsewhere. The result was a ‘waiting game’ while each hoped the other side would concede.
4 The Effect of Different Parameter Values on the Expected Time of Stabilisation

In this section, I am concerned with the effect of changing certain parameter values on the solution to the model.

4.1 Political Cohesion

In the model, the value of $\alpha$ is not bargained over. It is determined exogenously and both players know this value at the beginning of the game. It follows that if $\alpha = \frac{1}{2}$, stabilisation will occur immediately since there is nothing to be gained from delaying. This is because $V^W = V^L$ and since there are costs to not conceding, it is optimal to concede straight away. Conversely, where $\alpha$ is close to 1, there is an incentive to wait in the hope that the opponent will concede first. Therefore, the closer is $\alpha$ to 1, the larger is the delay, other things being equal.

This is an important result in terms of the ERM since it indicates the level of political cohesion within the community i.e. the willingness to share equally (or not) the burden of reserve depletion. Clearly, if there had been a great degree of political cohesion in 1992, a speculative attack on the Pound would have resulted in an immediate stabilisation with the UK and German governments sharing the burden. The fact that there was a delay in which the UK held out hoping for support indicates a lack of cohesion within the system. This then raises the issue of how political cohesion may be achieved. One possibility may be to require member governments to agree to share
equally any burden arising from a currency crisis. Clearly, this introduces the idea of precommitment and may strengthen the credibility of the ERM.

This has serious implications for a future attempt at monetary union. It suggests that unless there is a willingness to share the costs incurred, a speculative attack on a currency will merely lead to a repeat performance of the 1992 crisis.

4.2 Size of the Political and Financial Cost

Significantly, a change in the size of $c$ has no effect on the optimal time of concession. This is the point stressed by Casella and Eichengreen (1996). It is not an increasing burden which causes stabilisation. Instead it is generated by groups who do not know the ‘type’ of opponent they are facing. It becomes individually rational for each to hold out in the hope that the other has a value of $\theta$ larger than its own. Consequently, the size of the costs to the government following a speculative attack do not affect the optimal time of its concession. In considering future monetary union, this would suggest that the size of the total burden is not the issue. What is important is the share of this cost apportioned to each of the players.

5 Extension to the Framework

In the above analysis, I made an assumption that the exchange rate mechanism would survive the attack on the currency. The ‘war of attrition’ was concerned with who bears the larger share of the burden. In this extension to the framework, I shall consider the effect of country dependent payoffs
for the winner and loser. In particular, I shall assume that if the UK wins the ‘war of attrition’ then the system survives and the result is as before. However, if Germany is the winner, I shall assume that the UK leaves the system. This generates a lower payoff for both countries than in the previous scenario.

I assume that the pre stabilisation utility is (1) as before. However, following the stabilisation, the flow utility for Germany will be:

\[ u_g^L = -\alpha c \]  

(17)

if it loses or:

\[ u_g^W = -\gamma c \]  

(18)

where \( \gamma > 1 - \alpha \) if it is the winner. The payoff in being the winner is smaller than in the previous case since the UK has left the system. Conversely, the flow utility for the UK will be:

\[ u_{uk}^L = -\beta c \]  

(19)

if it loses, where \( \beta > \alpha \). This is also lower than in the initial case since, in this scenario, the UK will have to leave the exchange rate mechanism. If it wins the ‘war of attrition’ the flow utility will be:

\[ u_{uk}^W = -(1 - \alpha)c \]  

(20)

By including the discount rate in the above results one may arrive at
the corresponding discounted lifetime utilities at the point of stabilisation. The optimal times of concession for each country may now be calculated as before. The above results are substituted into (9). For the UK, this gives:

\[
\frac{dEU}{dT_i} = h(T_i)(\alpha + \beta - 1)\frac{c}{\delta} + [1 - H(T_i)] \left[c \left(\beta - \frac{1}{2} - \theta_i\right)\right] = 0 \quad (21)
\]

As before, I use (12), (13) and (21) to arrive at the differential equation:

\[
Tr(\theta) = \frac{-f(\theta)}{F(\theta)} \frac{\alpha + \beta - 1}{\delta (\theta + \frac{1}{2} - \beta)}
\quad (22)
\]

Solving (22) given the initial boundary condition of (15) gives:

\[
T(\theta) = \frac{\alpha + \beta - 1}{\delta (\theta + \frac{1}{2} - \beta)} \left(\ln \frac{\theta + \frac{1}{2} - \beta}{\theta + \frac{1}{2} - \beta} - \ln \frac{\theta - \theta}{\theta - \theta}\right) \quad (23)
\]

Firstly, if it is assumed that \(\theta > \beta - \frac{1}{2}\), then the government will concede in finite time. However, the main result here is that the optimal time of concession has increased. This is apparent when the term outside the brackets is examined. The value, \(\beta\), exceeds \(\alpha\). The significance of this is that the disparity between the winning and losing payoffs has increased. Therefore, there is a greater incentive to hold out in the hope that the opponent will concede.

For Germany, the opposite applies. Substituting into (9) gives:

\[
\frac{dEU}{dT_i} = h(T_i)(\alpha - \gamma)\frac{c}{\delta} + [1 - H(T_i)] \left[c \left(\alpha - \frac{1}{2} - \theta_i\right)\right] = 0 \quad (24)
\]

which, together with (12) and (13) gives:

15
\[ T_r(\theta) = -\frac{f(\theta)}{F(\theta)} \frac{\alpha - \gamma}{\delta(\theta + \frac{1}{2} - \alpha)} \]  

(25)

Solving for (25) with the initial boundary condition, (15) gives:

\[ T(\theta) = \frac{(\alpha - \gamma)}{\delta(\theta + \frac{1}{2} - \alpha)} \left( \ln \frac{\theta + \frac{1}{2} - \alpha}{\theta + \frac{1}{2} - \alpha} - \ln \frac{\theta - \bar{\theta}}{\bar{\theta} - \theta} \right) \]  

(26)

As in the initial case, an assumption that \( \theta > \alpha - \frac{1}{2} \) ensures that the government will concede in finite time. However, the important result here is that the optimal time of concession has decreased. The reason behind this is that the difference between the winning and losing payoffs has narrowed. Hence, the incentive to hold out for the opponent’s concession has been reduced. The greater the utility loss endured by the German government as a result of the UK leaving the system, the more likely is the German government to concede.

The final step in this analysis is to compare (23) with (26). The first point to note is that each government assumes that the opponent is playing the same strategy as itself. Hence it believes that the only way in which its opponent will differ will be in its value of \( \theta \). However, since each government receives a different flow utility according to whether it wins or loses, its optimal time of concession will also be governed by these factors. Hence, for a given value of \( \theta \), the closer is \( \gamma \) to \( 1 - \beta \), the smaller will be the difference between the countries’ optimal times of concession.

One may argue that this analysis is a more accurate description of the events of 1992 since the attack on the pound led to the UK leaving the exchange rate mechanism.
6 Conclusion

In conclusion, I would argue that this basic ‘war of attrition’ framework is extremely versatile. It lends itself, not only to the scenarios of foreign aid and tax distribution but also to the topical area of currency crises. My aim was to offer a possible explanation for the duration of a currency crisis and ultimately the timing of the UK exit from the regime. In the initial framework, it is assumed that the system remains intact. However, despite its simplicity, it has produced some interesting points with regard to political cohesion. Notably, in the case of the UK, there was a considerable delay until the decision was taken to leave the system. This delay would indicate a lack of political cohesion between countries in the ERM.

In the extension to this framework, I consider the effect of asymmetric payoffs for the winner and loser. I find that the larger the difference between the payoffs of winning and losing, the larger is the optimal time of concession.

I do not claim to have fully accounted for the events of the 1992 crisis. However, I have demonstrated, using a ‘war of attrition’ model, how the duration of the crisis may be explained. If each government knew its opponent’s ‘type’, then there would be no delay prior to a stabilisation. It is the fact that neither government knows the strength of its opponent, that creates a situation in which each finds it optimal to hold out in the hope that the opponent will concede first.
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