Optimal debt and equilibrium exchange rates in a Stochastic Environment: An Overview

Jerome L. Stein

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Optimal debt and equilibrium exchange rates in a Stochastic Environment:
An Overview

Jerome L. Stein
Division of Applied Mathematics Box F Brown University Providence RI 02912
Fax: (401) 863-1355. E-mail Jerome_Stein@Brown.edu

(This article is Chapter One of a forthcoming book by Jerome L. Stein entitled
STOCHASTIC OPTIMAL CONTROL, INTERNATIONAL FINANCE AND
DEBT CRISES, Oxford University Press)
This overview chapter explains in general terms the relevance and the contributions of this book to economic theory and policy. The economic theory and mathematics developed in chapters two and three derive *benchmarks* for the optimal debt in an environment where both the return on capital and the real rate of interest are stochastic variables. The equilibrium real exchange rate, the subject of chapter four, is where the real exchange rate is heading. These benchmarks are applied in chapters five through nine to answer the following questions.

- What is a theoretically based empirical measure of a "misaligned" exchange rate that increases the probability of a significant depreciation or a currency crisis?
- What is a theoretically based empirical measure of an "excess" debt that increases the probability of or a debt crisis?
- What is the interaction between an excess debt and a misaligned exchange rate?

Several historical examples indicate the significance of these questions. Then we sketch how the analytical tools developed in part II are applied in parts III and IV to answer these questions in a stochastic environment, where the return on capital and real interest rate are not predictable.

In July 1997, the economies of East Asia became embroiled in one of the worst financial crises of the postwar period. Yet, prior to the crisis, these economies were seen as models of economic growth experiencing sustained growth rates that exceeded those earlier thought unattainable. Similarly in 1998, the financial markets, the economics profession and the International Monetary Fund viewed Argentina as a model of stability and growth. In 2001-02 the Argentine economy defaulted on its huge debt.

Why did the financial markets, the International Monetary Fund, the World Bank and the bond rating agencies fail to anticipate the crises? In 2004, the International Monetary Fund Independent Evaluation Office IEO published a report that reviewed why and how, despite the Fund's extensive involvement with Argentina, the Fund was not able to help Argentina prevent and better manage the crisis. The primary purpose of the IEO evaluation is to draw lessons for the Fund in its future operational work.

The IEO report stated (pp. 22-23) that there is a general agreement that a combination of several external and internal factors contributed to the crisis: a weak
fiscal policy, a rigid exchange rate regime, and vulnerability to adverse external shocks. The IEO could not isolate the relative importance of these factors. "In the absence of the underlying vulnerability…the same adverse developments would not have had the catastrophic effects that were associated with the crisis, though they may well have produced some negative effects."

The factors underlying vulnerability must be given precise theoretical meaning with associated operational measures, to evaluate their explanatory power. The objective is to arrive at theoretically justified Early Warning Signals, based upon available information. The main reasons for the failures to anticipate balance of payments and debt crises were that the theories were based upon deterministic models, which ignored uncertainty, or that the theoretical tools were unduly limited in scope. For example, the most frequently used method to evaluate whether an exchange rate was misaligned was to compare the exchange rate with its Purchasing Power Parity (PPP) value. The PPP hypothesis assumes that the "equilibrium" real exchange rate is constant, but it does not provide a theory to explain what is the equilibrium real exchange rate. Moreover, this hypothesis lacks explanatory power. Empirical measures to estimate overvaluation compared the real exchange rate to its trend value. These eclectic empirical measures just add a trend to the PPP but cannot explain if an appreciation of the real exchange rate is a sign of strength or weakness in the balance of payments.

The most widely used measures of excess debt, which may lead to a debt crisis, focus upon two variables: (i) The ratio of debt/GDP that would result if current policies continued into the future, (ii) The trade balance/GDP that would keep the debt/GDP ratio equal to its current ratio. It is hypothesized that the higher the number, the more likely is it that there would be a debt problem. Empirical researchers concluded that these measures lacked explanatory power.

Since both measures of overvalued exchange rates and excess debt or debt burden were inadequate, the implied early warning signals were unreliable. A question that is relevant for policy is: what are theoretically based, operational Early Warning Signals that have explanatory power? One motive in writing this book is to answer this question.

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1 See Breuer (1994), MacDonald and Stein (1999) and Duval (2002).
2 See Kaminsky and Reinhart (1999).
Two theoretical tools are developed in this book. The analytical tool to estimate and explain the "equilibrium" real exchange rate is the NATREX model, an acronym for the natural real exchange rate. This is positive economics. The analytical tool to derive the optimal external debt/net worth and expected growth rate in an environment where both the productivity of capital and the real interest rate are unpredictable is stochastic optimal control dynamic programming (SOC/DP). This is normative economics. Both are benchmarks of performance. We then explain the interaction between misaligned exchange rates and excess debt that increases the probability of crises.

Equilibrium exchange rates and sustainable debts are not only relevant to the emerging markets, but also to the United States, the Euro area and to enlargement of the Euro area. The United States current account has been deeply in deficit in recent years. The growing negative net investment position leads to the question: how sustainable is the US current account deficit and associated inflow of capital? Is the US debt ratio excessive relative to the derived optimal debt ratio? What is the dynamic interaction between the real value of the dollar and current account deficits?

The real value of the Euro relative to the US dollar has fluctuated drastically since its inception. A frequently discussed question is whether the value of the US dollar/euro has been "misaligned", and what are the effects of policies in the U.S. or in Europe upon the exchange rate? A benchmark, the "equilibrium" real exchange rate, is required to answer this question.

The Central and Eastern European Countries CEEC are planning to join the European Monetary Union. These countries must establish the nominal values of their currencies upon entering Exchange Rate Mechanism, ERM-II. How should one evaluate the appropriateness of their nominal and real exchange rates? In the last ten years, the real values of their currencies measured in terms of tradable goods have been appreciating relative to the Euro. A correctly chosen exchange rate is a prerequisite for avoiding the depressing effects that occurred with the German reunification. An overvalued exchange rate hinders real growth, leads to sustained current account deficits and a large external debt. These factors could lead to either a debt crisis or a currency crisis. If the CEEC run into financial difficulties then, unlike the Eastern part of Germany which has been supported by the Western part, their debts will not be forgiven by the other members.
There is an explicit "no bail-out" clause in the Maastricht treaty (article 104b) that the CEEC signed when entering the EU. Moreover, if the exchange rate "disequilibrium" is sufficiently great, these countries may be forced to exit from the peg. An undervalued exchange rate would generate inflationary pressures that would violate the Maastricht criteria for entry into ERM II. We use the NATREX model developed in chapter four to evaluate what is an equilibrium exchange rate and to explain the appreciation of the real exchange rates of the CEEC: do they reflect strengths or are they Warning Signals of currency or debt crises? This question cannot be answered if "misalignment" is measured as the deviation of the real exchange rate from its trend.

The optimality analysis is based upon state of the art techniques of stochastic optimal control/dynamic programming (SOC/DP). The reasons for using these techniques are that optimization involves inter-temporal decisions. Current decisions not only affect current welfare, but they also have consequences for future welfare. The future is unpredictable, so that the optimal controls or decisions made at any instant should enter as feedback functions of the currently observable state.

The dynamic programming/stochastic optimal control techniques are widely used in the mathematical finance literature published in applied mathematics journals but are not widely used by economists. The stochastic optimal control techniques that we use to derive the optimal debt are quite technical. An attractive feature of our analysis of the optimal long term debt and expected endogenous growth is that we are able to show how the SOC/DP equations can be understood in terms of the Tobin-Markowitz mean variance M-V approach to portfolio selection. Thereby a relatively intuitive and graphic explanation - based upon the M-V techniques known to economists - can be given for the mathematical results.

**Summary of the theme and contributions**

The subject of this book is equilibrium real exchange rates, optimal external debt and their interaction. Our contributions can be summarized.

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4 Tobin developed the M-V analysis in 1952, several years before its publication, when I was a student in his graduate class in macroeconomics. That is why I dedicated the Fleming-Stein (2004) paper in his memory.
An explicit growth model is specified that explains how the real exchange rate and the external debt are affected by the exogenous and control/policy variables. A "story"/scenario is an integral part of the analysis.

Our equilibrium exchange rate, which is associated with internal and external balance, is called the natural real exchange rate (NATREX), because it is in the spirit of Wicksell's natural rate of interest. The medium run NATREX is a flow equilibrium, which is similar to the equilibrium concept used by Ragnar Nurske and John Williamson. The NATREX extends their work by developing the dynamics of capital and external debt. There is a trajectory from the medium run to the long run NATREX, where there are both flow equilibrium and stock equilibrium.

The equilibrium real exchange rate can be written as $R[Z(t)]$, where a rise is an appreciation and $Z(t)$ is a vector of measurable exogenous and control/policy real fundamentals that may vary over time. Misalignment $\Phi(t) = R(t) - R[Z(t)]$ is the difference between the actual real exchange rate $R(t)$ and the NATREX. Explicit empirical measures of "misalignment" and "excessive debt" are derived from the theory.

The actual exchange rate differs from the NATREX because of speculative, cyclical, and other ephemeral influences with zero expectations, but considerable variance. The real exchange rate converges to a band that contains the NATREX. Specifically, the trends in the NATREX explain the trends in the real exchange rate. This tells us which way the exchange rate is going. If measured misalignment overvaluation $\Phi(t) > 0$ is "sufficiently" large and sustained, a significant depreciation or a currency crisis is likely to occur. Similarly, if there is a significant undervaluation $\Phi(t) < 0$ and a pegged nominal exchange rate, then there will be significant inflationary pressure.

Currency and debt crises occur because actual behavior is not optimal. The subject of optimal debt, current account and endogenous growth concerns intertemporal decision making. The theoretical literature uses the Maximum Principle of Pontryagin or the Intertemporal Budget Constraint (IBC) to derive inter-temporal optimality conditions. The Maximum Principle is based upon perfect certainty. The trajectory to the

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5 See Driver and Westaway on concepts of equilibrium exchange rates.
6 Gandolfo and Turnovsky review the literature. Infante and Stein showed that this literature requires perfect knowledge and certainty, and showed that dynamic programming is a very much better method to derive inter-temporal optimality conditions.
steady state is unique, so that there is saddle point instability if there are any errors, however slight. The IBC constrains the present value of consumption to equal the present value of GDP over an infinite horizon. This literature is based upon certainty equivalence. Because the future GDP and interest rates are unpredictable, the present value of GDP over an infinite horizon, the IBC, is unknowable. The IBC is not operational and not enforceable. There is no feedback control mechanism to correct errors, which are certain to occur.

Instead, the techniques of stochastic optimal control/dynamic programming SOC/DP are used in this book to derive "inter-temporal optimization". We derive the optimal external debt/net worth, capital/net worth, consumption/net worth and the optimal endogenous expected growth rate in a stochastic environment.

The optimal debt/net worth $f^*$ or capital/net worth, derived from the SOC/DP analysis, is measurable for any arbitrary risk aversion. An excessive debt $\Psi_t = f_t - f^*_t$ is the deviation of the actual debt ratio $f_t$ from $f^*_t$ the optimal ratio. Generally the excess debt is produced by government budget deficits.

The greater is the measured excessive debt $\Psi_t$ the lower is the expected growth rate of consumption and the higher is its variance. It is therefore the more likely that random external shocks will lead to a debt default, rather than to a drastic decline in consumption.

The two types of crises are interrelated. A depreciation of the currency increases the real external debt burden, which raises the probability of a debt crisis. A debt burden adversely affects the current account and capital flows, which exert pressure on the exchange rate. We give precision to the concept of "vulnerability" to adverse developments on the basis of two theoretically based measures:

- **Excess debt** $\Psi_t = f_t - f^*_t > 0 \Rightarrow$ probability of debt crisis increases
- **Misalignment** $\Phi_t = R_t - R[Z_t] > 0 \Rightarrow$ probability of currency crisis increases;
- **Interaction** Probability of currency crisis $\Leftrightarrow$ Probability of debt crisis

_A guided tour_

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7 The work of Obstfeld and Rogoff (1996) is based upon the IBC.
A "guided tour" highlights some of our contributions with specific examples. It starts with the NATREX model of equilibrium exchange rates. A measure of misalignment is $\Phi_t$ derived based upon this model. The relation between the Purchasing Power Parity hypothesis and the NATREX is explained. An example shows how the NATREX model explains the medium to longer run movements in the real exchange rate of the Dollar-Synthetic Euro.

The second part of the guided tour is the analyses of both optimal short-term and long-term external debt. A measure of excess debt $\Psi_t$ is derived in each case. Early Warning Signals of a debt crisis, derived from the theoretical analyses, are given for emerging markets and Latin America. The discussion of the United States external debt and current account deficits, which is the subject of the last chapter, is not included in this guided tour.

Equilibrium Exchange Rates and Misalignment

An equilibrium exchange rate is where the exchange rate is heading. The concept and measure of the equilibrium exchange rate depends upon the time horizon and the underlying model. Several reasons have been cited in the literature why it is important to estimate equilibrium exchange rates. First, there are significant and sustained movements in exchange rates. For example, see figure 5.1 that graphs the U.S. dollar/euro exchange rate. These movements affect the competitiveness of the economies and their macroeconomic stability. One wants to know whether these movements are ephemeral or whether they are responding to "real fundamentals". This information is important because the answer has implications for rational macroeconomic policy and for rational investment decisions. If the depreciation of an exchange rate is due to a depreciation of its equilibrium value, then exchange market intervention or a restrictive monetary policy designed to offset the depreciation is counterproductive.

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8 It is not edifying to say that at every moment of time the exchange rate is determined by supply and demand, unless one can explicitly explain in terms of measurable variables what are their determinants and their evolution over time.
9 See for example the analytical survey article by Driver and Westaway.
10 The current controversy about the Chinese exchange rate revolves around the questions: what will be the effects of floating or a revaluation? What is the equilibrium exchange rate for China?
Second, in the case of monetary unions such as the Euro area, it is important to know how a potential entrant should select its exchange rate. An "overvalued" rate will depress growth and produce problems such as beset the eastern part of Germany. An "undervalued" rate will generate inflationary pressures. The measure of "over-valuation" and "under-valuation" must contain an explicit measure of an "equilibrium" real exchange rate.

Our emphasis is upon the equilibrium real exchange rate. It is defined as the nominal exchange rate times relative prices. In an adjustable peg regime, the nominal exchange rate is fixed and the actual real exchange rate varies due to changes in relative prices. In a floating exchange rate regime, both the nominal exchange rate and relative prices can lead to adjustments in the actual real rate. The only difference from our point of view is that the adjustment of the actual real exchange rate to the equilibrium will be faster when the exchange rate floats, because the nominal exchange rate is more flexible than relative prices.

A widely used approach in the literature is to "explain" the exchange rate by the uncovered interest rate parity theory (UIRP). It states that the anticipated appreciation of the exchange rate is equal to the anticipated interest rate differentials over a period of a given length. There are several limitations of this approach. First: the UIRP equation concerns the change in the exchange rate but does not contain any information concerning where the exchange rate is heading. As Driver and Westaway state, the exchange rate at any given time \( t \) will jump around to adjust to any change in either the anticipated exchange rate at some future date \( t+h \) or any change in anticipated interest rate differentials over the interval \( (t, t+h) \). The UIRP theory per se has no anchor.

Second: for the theory to have significance one must tie down the anchors. One anchor must be the "equilibrium" exchange rate and the second must be the path of the interest rates. This is not done in the UIRP theory.

Third: the theory states that the interest rate differential at time \( t \) is a good and unbiased predictor of the subsequent change in the exchange rate. The "tests" of the theory generally relate ex-post changes in the exchange rate to the previous interest rate

\[ 11 \text{ See MacDonald (1999, pp. 36-38) and Driver and Westaway (section 4.1) for critiques of this theory. See Stein and Paladino (1997) for a critique of the literature.} \]
differentials. In general, the results of these tests are not encouraging. The interest rate differential has the incorrect sign and is unsuccessful in predicting exchange rate movements\textsuperscript{12}.

For these reasons, authors who are interested in explaining exchange rates focus upon the anchor, the equilibrium exchange rate - where the exchange rate is heading. Then the theory of UIRP has structure. The actual exchange rate at time \( t \) is equal to the present value of the equilibrium exchange rate, where the discount factor is the interest rate differential. There are two types of candidates for the equilibrium exchange rate. One is Purchasing Power Parity (PPP), which assumes that the equilibrium real exchange rate is a constant. As mentioned above, this hypothesis is unimpressive as an explanation of the anchor\textsuperscript{13}.

The other candidate is an equilibrium real exchange rate that depends upon time varying real, measurable "fundamentals"\textsuperscript{14}. This has led to the literature of " equilibrium exchange rates", which was given great impetus by John Williamson's influential book (1994). The logic of this approach goes back to Ragnar Nurske's article. The "equilibrium" exchange rate is the exchange rate that is associated with both external and internal balance. Anticipations, speculative capital movements and changes in reserves are excluded from the concept of an equilibrium real exchange rate, which is where the exchange rate is heading. The NATREX model of equilibrium exchange rates generalizes the work of Williamson and Nurske. It is a Neoclassical growth model, whose underlying equations are based upon intertemporal optimization by the private sector, but not the government whose decisions are political.

The NATREX explains the fundamental determinants of the medium run equilibrium and the dynamic trajectory to the long run equilibrium. In the medium run equilibrium there are both internal and external balance. In both the medium run and longer run the NATREX equilibrium real exchange rate satisfies equation (1), subject to constraints. The constraints are that there is internal balance, where the rate of capacity utilization is at its longer term mean, and external balance where the real rates of interest

\textsuperscript{12} A large literature is devoted to "rationalizing" the failure of the UIRP theory but does not test their conjectures in an objective manner.

\textsuperscript{13} A clear summary of the failure of the PPP and evaluation of the econometric work is in Breuer (1994, esp. pp. 273-74).

\textsuperscript{14} This is the subject of the MacDonald (1999) article.
at home and abroad are equal, there are neither changes in reserves, nor speculative capital flows based upon anticipations. The equilibrium real exchange rate is the mean of a distribution, which is based upon real fundamentals. The mean will vary over time due to endogenous changes in capital and external debt, as well as changes in the exogenous real fundamentals. Deviations from this mean are produced by speculative factors involving anticipations, by cyclical factors, lags in adjustment, and interest rate differentials. These disequilibrium elements average out to zero. These deviations produce considerable variation but their effects are ephemeral.

The terms in square brackets are that investment less saving \((I_t - S_t)\) plus the current account is equal to zero. Investment less saving is the non-speculative capital inflow. The current account \((B_t - r_tF_t)\) is the trade balance \(B_t\) less transfers of interest and dividends \(r_tF_t\). The net external debt is \(F_t\) and \(r_t\) is the "interest/dividend" rate. The international investment position consists of equity, portfolio investment and direct investment. The debt \(F_t\) is the negative of the net international investment position. Measure investment, saving and the debt as fractions of the GDP.

\[
(I_t - S_t) + (B_t - r_tF_t) = 0
\]

All of the authors who take the equilibrium real exchange rate approach use equation (1) to determine the exchange rate. The main differences among them concern their treatment of the two terms. Some work with a concept of what is a "sustainable" current account such that the debt does not "explode". As is discussed in the chapter on the United States current account deficits, their estimates are subjective, so their equilibrium exchange rate is a "normative" concept. The NATREX approach is quite different in several respects, primarily because the endogenous current account generates an evolving external debt, which feeds back into the medium run equation (1). A trajectory to longer run equilibrium is generated. The other difference is that the underlying equations are derived from inter-temporal optimization by the private sector.
The dynamics of the debt/GDP ratio $F_t$ is equation (2), where $g$ is the growth rate. The current account deficit is the change in the external debt. The real exchange rate affects the trade balance $B$ in equation (1), and the trade balance affects the evolution of the actual debt ratio in equation (2). There is a dynamic interaction between the endogenous real exchange rate and debt ratio.

\[ (2) \quad \frac{dF_t}{dt} = (I_t - S_t) - g_t F_t = (r_t F_t - B_t) - g_t F_t = (r_t - g_t) F_t - B_t \]

In longer run equilibrium, the debt ratio stabilizes at a value that satisfies equation (3). The trade balance $B_t$ is sufficient to finance the interest plus dividend transfer on the debt net of growth $(r_t - g_t)F_t$. A negative debt is net foreign assets.

\[ (3) \quad (r_t - g_t) F_t - B_t = 0. \]

The longer-run equilibrium real exchange rate $R_t^*$ and debt/GDP ratio $F_t^*$ are endogenous variables that satisfy both equations (1) and (3). They are written as (4) and (5) to indicate that they both depend upon the real fundamentals $Z_t$.

\[ (4) \quad R_t^* = R(Z_t) \]
\[ (5) \quad F_t^* = F(Z_t). \]

We call dynamic stock-flow model equations (1) - (3) the NATREX model, which is an acronym for the Natural Real Exchange Rate\textsuperscript{15}. This is a model of positive economics. The literature associated with Williamson's FEER uses equation (1) and does not contain the dynamic interactions, equations (2) and (3). The NATREX model derives the private saving, private investment and trade balance equations from optimization criteria. There is no presumption that the government saving and investment decisions are optimal, since they are based upon political considerations not upon social welfare.

**Populist and Growth Scenarios**

The NATREX model is a technique of analysis\textsuperscript{16}. The purpose of the model is to understand the effects of policies and external disturbances upon the trajectories of the equilibrium real exchange rate $R_t$ and equilibrium debt ratio $F_t$, which depend upon the vector of fundamentals $Z_t$. Insofar as the fundamentals vary over time, the equilibrium real exchange rate and debt ratio will vary over time, as indicated in equations (4) and

\textsuperscript{15} The NATREX appellation was suggested by Liliane Crouhy-Veyrac who compared the model to Wicksell's "natural" rate of interest.

\textsuperscript{16} Allen explains the flexibility of this method of analysis.
The logic and insights of the NATREX model can be summarized in two scenarios. Each scenario concerns different elements in the vector $Z_t$ of the fundamentals, and has different effects upon the equilibrium trajectories of the real exchange rate NATREX and of the external debt. NATREX analysis concerns the equilibrium real exchange rate and it is neither the actual real exchange rate nor the optimal exchange rate that would lead to the optimal debt ratio.

The first scenario, called the Populist scenario, involves a decline in the ratio of social saving/GDP. This could occur when the government incurs high-employment budget deficits, lowers tax rates that raise consumption, or offers loan guarantees/subsidies for projects with low social returns. This represents rise in the consumption ratio/a decline in the saving ratio, a shift in the S function in equations (1) and (2). These Populist expenditures are designed to raise the standards of consumption/quality of life for the present generation.

The second scenario, called the Growth scenario, involves policies designed to raise the productivity of capital. Policies that come to mind involve the liberalization of the economy, increased competition, wage and price flexibility, the deregulation of financial markets, improved intermediation process between savers and investors, and an honest and objective judicial system that enforces contracts. Growth policies improve the allocation of resources and bring the economy closer to the boundary of an expanding production possibility curve.

Table 1 summarizes the differences between the two scenarios in the medium and the long run. The stories behind the dynamics are as follows.

The Populist scenario involves increases in social (public plus private) consumption relative to the GDP. External borrowing must finance the difference between investment and saving. The capital inflow appreciates the real exchange rate from initial level $R(0)$ to medium run equilibrium $R(1)$, where $T = 1$ denotes medium run equilibrium. The current account deficit is balanced by the capital inflow. The debt rises, since the current account deficit is the rate of change of the debt - equation (2). Current account deficits lead to growing transfer payments $rF_t$. This Populist scenario is potentially dynamically unstable because the increased debt raises the current account.
deficit, which then increases the debt further. The exchange rate depreciates, and the debt rises, steadily.

Stability can only occur if the rise in the debt, which lowers net worth equal to capital less debt, reduces social consumption/raises social saving. For example, the growing debt and depreciating exchange rate force the government to decrease the high employment budget deficit. Thereby, saving less investment rises. Long-run equilibrium (denoted by $T = 2$) is reached at a higher debt $F(2) > F(0)$ and a depreciated real exchange rate $R(2) < R(0)$. The longer-run depreciation of the exchange rate $R(2) < R(0)$ can be understood from equation (3). The debt is higher than initially. Therefore, the trade balance $B(2)$ must be higher than initially to generate the foreign exchange to service the higher transfers$^{17}$. The real exchange rate must depreciate to $R(2) < R(0)$ in order to raise the trade balance to $B(2)$.

Table 1

NATREX dynamics of exchange rate and external debt: Two Basic Scenarios

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Medium-run, $T = 1$</th>
<th>Long-term, $T = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Populist:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rise in social consumption (discount rate, time preference), rise in high employment government budget deficit, decline social saving</td>
<td>appreciation $R(1) &gt; R(0)$</td>
<td>depreciation $R(2) &lt; R(0) &lt; R(1)$</td>
</tr>
<tr>
<td>Debt rises</td>
<td>$F(1) &gt; F(0)$</td>
<td>Debt rises</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F(2) &gt; F(1) &gt; F(0)$</td>
</tr>
<tr>
<td><strong>Growth oriented:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rise in productivity of investment, expansion of production possibility set. Rise in growth, rise in competitiveness</td>
<td>appreciation $R(1) &gt; R(0)$</td>
<td>appreciation $R(2) &gt; R(1) &gt; R(0)$</td>
</tr>
<tr>
<td>Debt rises</td>
<td>$F(1) &gt; F(0)$</td>
<td>Debt declines</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F(2) &lt; F(0) &lt; F(1)$</td>
</tr>
</tbody>
</table>

$^{17}$ The interest rate must exceed the growth rate if the expected present value of future income is finite.
The *Growth scenario* is summarized in the lower half of table 1. The perturbation is a rise in the productivity of investment and an expansion of the production possibility set. Investment rises because of the rise in the rate of return. The difference between investment and saving is financed by a capital inflow. The exchange rate appreciates to \( R(1) > R(0) \) which reduces the trade balance and produces a current account deficit. The initial current account deficit equal to \([I(0) - S(0)]\) raises the debt. The trade deficit provides the resources to finance capital formation, which raises the growth rate and the competitiveness of the economy.

It does not matter much where the rise in the return on investment occurred or what factors led to an expansion of the production possibility set. If they are in the traditional export or import competing sectors, the *trade balance function* \( B = B(R;b) \) increases. The B function, which relates the real value of the trade balance to the real exchange rate \( R \), increases with a rise in the overall productivity of the economy. For example, the reallocation of resources leads to the production of higher quality/value goods that can compete in the world market. If the rate of return on investment and productivity increase in the sectors that are not highly involved in international trade, resources can then be released for use in the more traditional "tradable" sectors. Again, the B function supply curve increases.

The trajectory to longer-run equilibrium differs from that in the Populist scenario. The crucial aspect implied by the Growth Scenario is that, at medium run equilibrium exchange rate \( R(1) \), *the trade balance function increases*. The real exchange rate appreciates and there are now current account *surpluses*, excess of saving over investment. As a result, the debt then declines to a new equilibrium \( F(2) < F(0) \). The trajectory of the debt is not monotonic. The net effect in the longer-run can be understood from equation (3). The debt is lower, the growth rate is higher and the trade balance function B has shifted to the right. *The long-run equilibrium exchange rate must appreciate to reduce B to equal the lower value of \((r-g) F^*\).*

The dynamic process in the Growth scenario is *summarized* in the lower half of table 2. The *real exchange rate* appreciates steadily to a higher level \( R(2) > R(1) > R(0) \). The *external debt* reaches a maximum and then declines to \( F(2) < F(0) < F(1) \).
The Nominal Exchange Rate: PPP and the Equilibrium Exchange Rate Models

The most frequently used estimate of the equilibrium nominal exchange rate is based upon the Purchasing Power Parity (PPP) hypothesis. The PPP arbitrarily assumes that the equilibrium real exchange rate is a constant. PPP cannot and does not purport to explain what determines the equilibrium exchange rate, what are the effects of policy/control variables and exogenous variables upon the equilibrium real exchange rate. Hence it is not particularly useful for policy questions. For example, PPP is unable to answer the following significant questions: At what exchange rate should the CEEC enter the Euro area, to avoid the problems that occurred with the integration of East Germany? What policies will be consistent or inconsistent with the established exchange rates to avoid deflationary or inflationary pressures? How can one explain the trends in the values of the Euro and the US dollar?

The "equilibrium exchange rate" literature\(^\text{18}\) takes a very different point of view. The NATREX model implies that one would observe PPP in the long-run only if \(R(Z_t)\) in equation (4), a linear combination of the fundamentals, is mean reverting in the longer run. The PPP model is a special case of the NATREX model. The relation between the PPP and the equilibrium exchange rate models can be understood from figure 1 and equation (6). The logarithm of the equilibrium nominal exchange rate\(^\text{19}\) denoted \(\log N^c_t\) has two components: the logarithm of the equilibrium real exchange rate, \(\log R[Z_t]\) which is the NATREX, and the logarithm of the ratio of relative domestic/foreign "prices"\(^\text{20}\) denoted \(\log \left[ \frac{p_t}{p^*_t} \right] \). The PPP ignores the \(R(Z_t)\) term by assuming that it is a constant, and focuses exclusively upon the relative price term. The NATREX is not a constant, but varies with the vector of fundamentals \(Z_t\) that underlie the saving, investment and trade balance functions.

\[
(6) \quad \log N^c_t = \log R[Z_t] - \log \left[ \frac{p_t}{p^*_t} \right].
\]

Figure 1 describes three values of \(R(Z)\), where \(R(1)\) is the most appreciated NATREX, \(R(2)\) is the most depreciated value and \(R(0)\) is the mean NATREX. Suppose

---

\(^\text{18}\) This is the approach taken by Williamson, Clark and MacDonald among others. NATREX is in the set of these models.

\(^\text{19}\) A rise in the nominal or real exchange rate is an appreciation of the currency.

\(^\text{20}\) The best choices are either relative GDP deflators or relative labor costs.
that $Z = 0$ and the corresponding *equilibrium real* exchange rate NATREX is $R(0)$. Then the equilibrium *nominal* exchange rate is a set of points along line $R(0)$. The PPP relation would hold as long as the NATREX remained constant. If the nominal exchange rate were above the line $R(0)$, the currency is overvalued. There cannot be internal and external equilibrium. The country would have difficulty competing in international markets. It would either lose reserves and the external debt/GDP ratio would rise, or there would be depressed economic conditions, particularly if the monetary/fiscal authorities attempt to stem the excess demand for foreign exchange. Similarly, if the nominal exchange rate were below the line, then reserves would rise or there would be inflationary pressures. Nominal exchange rates either above or below the line are unsustainable. Either the nominal exchange rate or relative prices must adjust, if both internal and external equilibrium are to prevail.

Figure 1. The equilibrium nominal exchange rate the NATREX and relative prices. If the NATREX varies between $R(1)$ and $R(2)$, and relative prices vary between $c$ and $d$, the equilibrium nominal exchange rate will be contained in the rectangle.
The NATREX changes with the fundamentals vector $Z_t$, as described in the two scenarios summarized in section table 2. As the NATREX varies between $R(1)$ and $R(2)$ and relative prices vary between $c$ and $d$, the equilibrium nominal exchange rate will be contained in the rectangle. A regression of the nominal exchange rate upon relative prices would be based upon the scatter of points in the rectangle. If the relative prices are constant at $\log \left[ p_t/p^*\right] = 0$, then the equilibrium nominal exchange rate varies from $a$ to $b$. If the nominal exchange rate is fixed at $\log N_t = 0$, then relative prices must vary between $c$ and $d$.

Example: The Euro - United States Dollar Exchange Rate

With the introduction of the Euro in 1999, there was a multitude of predictions concerning its future value relative to the U.S. dollar. Most of the predictions were that the Euro would appreciate - be a "strong currency" - as institutions diversified their portfolios away from the dollar. However, the Euro depreciated from $1.16$ in January 1999 to $0.87$ in February 2002. Then the predictions switched towards pessimism about the value of the Euro. However, the Euro appreciated to $1.34$ in December 2004. This appreciation caused consternation because it adversely affected the competitiveness of the European economies. Pressures were exerted upon the European Central Bank to offset the appreciation with an expansionary monetary policy. In June 2005, the Euro was trading at around $1.20$.

The standard theories were not able to explain these trends. As explained in the Preface, researchers in Europe examined to what extent the NATREX model could explain the movements in the Euro. In particular, the staff of the European Central Bank had to decide whether the euro was or was "misaligned". The strategy of the researchers was to construct a "synthetic euro", which is a weighted average of the currencies of the countries in the Euro area, from the beginning of floating in the 1970s to the advent of the Euro. Chapter five is devoted to an evaluation of research concerning the Euro exchange rate, and compares the NATREX explanation with other approaches.

The NATREX model states that the fundamental determinants of the real exchange rate $R(Z_t)$ and the debt ratio $F_t$ are the variables $Z_t$ in table 1: relative social
consumption ratio, relative productivity of investment, and relative productivity of labor in the pair of countries considered. The economic explanation of how these fundamentals affect the NATREX is the two scenarios. The signs of their medium run and long run effects are specified in table 1.

The model in equations (1)-(4) has been tested and applied in several ways. The structural equation approach estimates the components, saving, investment, and trade balance equations to obtain the medium run equilibrium NATREX from equation (1). Then, that solution is used in the dynamic equation (2) to obtain the long run NATREX from equation (3). The reduced form approach just concentrates upon the dynamics of the real exchange rate. There is an excellent correspondence between the implied dynamic process and the Vector Error Correction econometric approach. The long run NATREX equation (4) is the hypothesized cointegrating equation.

Using the fundamentals Zt for the Euro area relative to the US, one obtains an estimate of the NATREX labeled R(Z). The coefficients have the signs specified in the model, table 1. Thus we have an estimate of the equilibrium real exchange rate. Adding the relative price variable, we obtain an estimate of equation (6)/figure 1 for the equilibrium nominal value of the Euro. A rise is an appreciation of the Euro or a depreciation of the United States dollar.

Figure 2 graphs the actual nominal value of the synthetic euro (EUUSNERMA = $US/euro) and the estimate of the equilibrium nominal value (NOMNAT), based upon the NATREX model. The NATREX is a model of the equilibrium exchange rate, not the actual exchange rate. The actual exchange rate is hypothesized to converge to a distribution whose mean is the equilibrium exchange rate. The equilibrium rate varies according to figure 1/equation (6) - because there are both shifts in the R(Z) curve as well as movements along the curves due to relative prices. Since the equilibrium nominal exchange rate varies with both vector Zt and relative prices (pt/pt*), price stability alone is not a sufficient condition for exchange rate sustainability.
Figure 2. Nominal value synthetic euro (4 Q MA), $US/Euro = EUUSNERMA; NATREX estimate, equation (6) is NOMNAT, 1970 to 2000.

Figure 2 shows the undervaluation of the synthetic euro (the overvaluation of the $US) in the first half of the 1980's, and in the period after 1996. Estimates of the equilibrium value of the Euro from 1999 - 2001 indicated that it was undervalued relative to the $US. This estimate is consistent with the significant appreciation of the Euro since 2001. The euro appreciated from $0.85 in 2001 to $1.29 in November 2004.

The NATREX model is a useful guide to policy. Insofar as the fundamentals Z - relative social consumption ratios, relative productivity and relative returns on capital - have not changed drastically, the NATREX real exchange rate should be relatively constant. Insofar as relative prices also have not changed much, the NATREX nominal exchange rate should be relatively constant at about $1.20 in figure 2. This is an
equilibrium exchange rate. Speculative forces based upon anticipations, political events such as referenda, cyclical effects and monetary policies will produce deviations from the equilibrium. However, these are transitory effects that should wash out. Exchange market intervention, which attempts to drive the exchange rate from the NATREX will be ineffective for the reasons given above.

This explanation of the Euro rate is the subject of chapter five, and a similar analysis is developed in chapter six for the exchange rates of the transition economies in Eastern Europe.

Optimal Debt Models

Many studies conclude that external borrowings, particularly by commercial banks and firms, were among the key factors responsible for the severity of the East Asian crises in the late 1990s. In particular, the authors argue that short-term capital flows are volatile and hence the size of foreign currency denominated debt contributes significantly to the occurrence of currency and debt crises. The volatility of exchange rates is strongly affected by the stock of external debt.

Countries have both short-term and long-term debt. The inter-temporal optimization problem arises because the debt occurred to finance consumption and investment at one time affects the consumption possibilities at a later date. This choice is seen in balance equations (7) and (8). The optimal amounts of debt differ according to whether it is short-term or long-term.

In the case of short-term investment if the borrowing is at time $t$ then it must be repaid with interest at later date $s = t + \Delta t$. In the case of long-term investment, the debt does not have to be repaid at any terminal date, but must be serviced regularly. Equation (7) describes the change in the debt $dL_t$. External debt $L_t$ rises because consumption $C_t$ plus investment $I_t$ plus the debt service $r_tL_t$ exceeds $Y_t$ the GDP. Alternatively, the change in the debt is $(I_t - S_t)dt$, investment $I_t$ less saving $S_t = Y_t - r_tL_t - C_t$ over the period of length $dt$.

(7) $dL_t = (I_t - S_t)dt = (C_t + I_t + r_tL_t - Y_t)dt = \text{current account deficit}$
The actual debt is generated by the decisions of the government and the private sectors. Fiscal and monetary policies are important determinants of investment and social saving by the private plus public sectors. In the Latin American countries the debt has risen due to high consumption and/or low social saving by the public plus the private sectors. In the Asian countries industrial policy stimulated private investment. The excess of investment less saving led to a capital inflow and an increase in the external foreign currency denominated debt. There is no presumption that these decisions - which produced the capital inflow $dL_t$ - have been optimal, as is clear from the recurrence of debt and balance of payments crises in South-East Asia and Latin America.

The external debt has to be serviced and that would clearly affect future consumption. We can see this by writing consumption at some later time $s = t + dt$, in equation (8) below. It is another way of writing equation (7), but at a later date. Consumption is equal to the GNP, which is equal to the GDP less the debt service $Y_s - r_s L_s$, less investment $I_s$ plus new borrowing $dL_s$. The new borrowing is the net capital inflow in the form of either direct investment, portfolio investment or short term bank flows.

$$ (8) \ C_s \ ds = (Y_s - r_s L_s - I_s) ds + dL_s $$

It is important to have a benchmark, what is the optimal debt, so that we can compare the actual to the optimal debt. Thereby we would obtain an Early Warning Signal, $\Psi_t = f_t - f^*_t$ above of a crisis. Three elements must be specified to solve the inter-temporal optimization problem. Different models involve different specifications of these three elements.

- The constraints and controls/policy decisions.
- The dynamic stochastic process
- The optimality criterion

In chapter two, the debt is modeled as short-term corresponding to bank loans. In the model discussed in chapter three, the debt is long term, which would correspond to direct investment or long-term portfolio investment. The major theoretical difference concerns the constraints, so that the mathematical analysis is very different in each case. They must be modeled differently, and one case cannot be modeled as a special case of the other, for the following reason.
In the short-term debt model sketched below, debt is incurred in period $t=1$ which has a maturity at period $t=2$. It is a repeating two period model. There are several constraints. First: at maturity, the debt must be repaid with interest. Second: the capital at the beginning of period $t=3$ must be the same as it was at the beginning of $t=1$, so the process is repeated. Third: consumption in period $t=2$, when the debt is repaid, must exceed a certain minimum - regardless of the state of nature. This is a "no bankruptcy" constraint. The argument is that, if the attempt to service the debt would reduce consumption below the minimum - which we arbitrarily set at zero - then the country would default. Faced with a choice: (a) repay debt and drastically reduce the standard of living, or (b) default but do not drastically reduce the standard of living, the economy would select the second option. The controls/policies are the consumption, investment and resulting debt in period $t=1$, subject to the constraints.

In the long-term model of chapter three, sketched below, there is no maturity to the debt but it must always be serviced. It is an infinite horizon model in continuous time. The controls - debt, capital and consumption - are constantly adjusted to keep these control/policy variables at their derived optimum levels. The constraints are that: consumption always be positive, regardless of the state of nature, and that net worth is always positive. The latter avoids Ponzi schemes, where new borrowing is used to service a growing debt. Inter-temporal optimization in the long-term model involves the use of dynamic programming.

In both cases, a debt crisis is produced when the actual debt significantly exceeds the constrained optimal debt. The actual debt is generated by saving and investment decisions by the private and public sectors, which may be far from optimal. The economy is more vulnerable to external shocks when the actual debt significantly exceeds the constrained optimal debt.

The second specification concerns the stochastic process. We model the two sources of uncertainty that ultimately affect consumption. The first source of uncertainty is the ratio of GDP per unit of capital $Y_t/K_t$ and the second source is the real rate of interest $r_t$. Two stochastic variables real GDP and real interest rate, which will affect consumption at the later date, are written in bold letters in equation (8). Each one is highly variable. If bad shocks reduce the GDP and raise real interest rates, and investment
falls to a minimum level then consumption in equation (8) may have to be reduced - unless there is new borrowing to offset the decline.

The output/capital ratio $Y_t/K_t = b_t$ has a deterministic component $b$, which is the mean return on capital, and a stochastic component with a zero mean and a significantly positive variance. The deterministic component $b$ corresponds to the slope of a regression of the growth of GDP on the ratio of investment/GDP, and the stochastic part corresponds to the standard error of estimate. This stochastic part contains the "Solow residual", variations in the rate of capacity utilization resulting from fiscal and monetary policies, variations in the terms of trade and the composition and quality of the investments.

The second source of uncertainty concerns the real interest rate $r_t$ required to service the external debt $L_t$. For countries other than the US - such as emerging market countries - the real interest rate in terms of consumer goods $r_t$ has three components. The first is the interest rate on US Treasury long-term debt. The second is the premium on dollar denominated debt charged to sovereign borrowers. The third is the anticipated exchange rate depreciation of the currency. A currency depreciation increases the amount of consumer goods that must be sacrificed to service/repay the foreign currency denominated debt. The equation for the real interest rate contains two terms: the first term is deterministic with a mean real rate of interest $r$ and the second term is stochastic with a positive variance.

The expectations of the stochastic terms are equal to zero, but the productivity of capital and real rate of interest may be correlated. In developed countries such as the United States and Europe, the correlation is generally positive. In periods of rapid growth, there is a rise in investment demand and demand for money; and interest rates rise. When there are financial crises, such as occurred in Asia or Latin America, the growth of GDP and real interest rate are negatively correlated, for the following reason. A decline in GDP may occur because of a decline in the terms of trade and/or the anticipated return on investment turns out to be an illusion and the asset bubble collapses. The stochastic term in the productivity of capital equation is negative. Since firms borrow primarily from the banks to finance real investment and the banks in turn primarily finance their loans by borrowing US dollars in the international capital market,
a domino effect is created in the event of a financial panic. When debtors are unable to repay their loans to the banks, the banks in turn are unable to repay their loans to international creditors. Financial panic leads to a short term capital flight. The government may try to stem the outflow by using the dollar reserves, but that is only a stopgap measure. Sooner or later the monetary authorities will raise interest rates and, when that fails to stem the outflow, the currency will depreciate. The depreciation of the currency implies that the real rate of interest to repay a debt denominated in foreign currency rises. In that event, the stochastic term in the real interest rate equation is positive. The situation is exacerbated when banks also denominate their loans to the domestic firms in US dollars. Firms would find it very difficult to service debts denominated in foreign currency because they are faced with both a rising nominal rate of interest and a depreciating currency. A negative correlation between the productivity of capital and the real rate of interest makes an external debt very risky.

Faced with these sources of uncertainty, how then should a country select its optimal debt and level of consumption? The third specification concerns the optimality criterion. One criterion is that the controls are selected to maximize the expectation of the discounted value of a concave utility of consumption over the appropriate horizon. A second criterion is that the debt and capital are selected to maximize the expected value of the growth rate of consumption over a horizon, subject to the constraint that the ratio of consumption/net worth is a positive constant. A third criterion is a very conservative one. The controls are selected to maximize the minimum expected value of the utility of consumption. Only the first two criteria are used in this book.

Short-debt model in Discrete Time Finite Horizon

For many countries, short-term capital flows are important in financing investment less saving and have been associated with crises, such as in Southeast Asia 1997-98. In this part, we sketch the derivation of the optimal investment, consumption and debt in the short-term capital movements model, which is the subject of chapter two. Explicit equations for excess debt $\Psi_t$ and Early Warning Signals of a debt crisis are stated. We provide specific examples of how this analysis can explain the default risk in

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emerging market countries and Latin America. Detailed empirical application of the Short-debt model is the subject of chapter seven.

The model assumes two repeating discrete time periods. In period one, the country has a stock of capital $K_1$ and a Gross Domestic Product $Y_1$. The controls are consumption $C_1$ and investment $I_1$. If consumption plus investment is greater than the GDP, the country incurs an external debt $L_1$ to finance the difference. If consumption plus investment is less than the GDP the country is an international creditor, and the debt $L_1$ is negative. The debt, or net foreign assets, bears a known real rate of interest. At the second period, the debt plus interest must be repaid. We consider a repeating two period model, so that the capital at the beginning of period three must be the same as it was at the beginning of period one. This constraint means that the sum of investment over the two periods must be zero.

The productivity of capital $Y_t/K_t = b_t$ is a stochastic variable. When the investment decision $I_1$ is made in period one, the productivity of capital in period two $b_2 = Y_2/K_2$ is unknown. Capital in period two is the capital at the beginning of period one plus the investment made in period one. Two possibilities are considered. Either the productivity of capital in period two $b^+$ exceeds the interest rate $r$, with probability $1 > p > 0$, or the productivity of capital $b^-$ is less than the rate of interest with probability $(1-p)$.

The debt in period one $L_1$ finances investment $I$ less saving $S$. The stochastic variable $b_2$ is written in bold letters. Consumption in period two $C_2$ is equal to the GDP in period two $Y_2 = b_2K_2 = b_2(K_1 + I_1)$ less the repayment of the debt plus interest $(1+r)L_1$ plus the disinvestment to make capital at the beginning of period three equal to the initial capital $K_1$. Equation (9) describes consumption $C_2$ in period two. Since the return on capital can assume two values: $b^+ > r$ in the good case, and $b^- < r$ in the bad case, consumption in period two can assume either $C_2^+$ in the good case or $C_2^-$ in the bad case.

\[
C_2 = b_2K_1 + [(1+r)(b_1K_1 - C_1)] + (b_2 - r)I_1.
\]

There are three components to consumption in period two, equation (9). If there is neither saving $(S_1 = b_1K_1 - C_1)$ nor investment in period one – if consumption is equal to GDP in the first period – then consumption in period two would just be the GDP in period two $Y_2 = b_2K_1$.

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22 This assumption is relaxed in the long term optimal debt model.
If there is saving but no investment in period one, then consumption in period two is the sum of the first two terms. The saving is invested abroad at the known rate of interest, and permits the economy to consume \([(1+r)(b_1K_1 - C_1)]\). This term is not stochastic.

If there is investment in period one, then the additional consumption available in period two is the stochastic net return times the investment - the third term \((b_2 - r)I_1\).

If the bad state of nature occurs \(b_2 = b_2 - r\) then the burden of the debt resulting from \((b - r)I_1\) could depress consumption \(C_2\) to an intolerable level. In that case, the country would default rather than accept the required reduced standard of living.

The constrained optimization decision is to select the controls: consumption \(C_1 > 0\) and investment \(I_1 \geq 0\) during period one to maximize the expectation over the stochastic variable \(b_2\) of the discounted value of utility over the two periods. We assume a HARA utility function, \(U(C) = (1/\gamma)C^\gamma\), with positive risk aversion \((1-\gamma) > 0\). A special case that we use frequently is \(\gamma = 0\), so that the utility function is logarithmic \(U(C) = \log C\). The great advantage of using the HARA function, particularly in the long-term model in chapter three, is that one can solve for the optimal controls analytically. Otherwise, the optimal controls are solved numerically by using a computer.

An important constraint is that there should be no default. This means that consumption in period two, in the bad case, should exceed a minimum tolerable level \(C_2 > C_{\min} \geq 0\).

The solution of the Short-debt model is the subject of chapter two. The conclusions are described in figure 3 for the optimal debt/capital \(f = L_1/K_1\). Concentrate
upon the logarithmic case, with risk aversion equal to unity, where the results are clear

\[ f = \frac{L}{K} \]

\[ \text{Optimal debt/capital } f = \frac{L}{K} \]

Figures 3. Optimal debt/capital \( f = \frac{L}{K_1} \) is curve ABDEF.

Expected net return \( x = \mathbb{E}(b - r) = \left[ pb^+ + (1-p)b^- \right] - r \). Along ABD the country is a creditor. Along DEF the country is a debtor. If debt/capital exceeds \( f_{\text{max}} \), the probability of default is \((1-p)>0\).

Optimal saving/capital is a constant independent of the expected net return \( x = \mathbb{E}(b - r) \). Optimal investment/capital is zero for expected net return \( x < a \) in figure 3. Risk premium \( a \) is related to the ratio of the possible loss from investment in capital relative to the return if all wealth were invested abroad at the safe return. This means that, for \( x < a \), the country will be a creditor and will invest all of its saving abroad earning the safe rate of return \( r > 0 \). The debt/capital will be \( f_{\text{min}} < 0 \), where the country is a creditor.

When the expected net return rises above \( a \), investment will be positive, thereby reducing the capital outflow. When the expected net return \( x = D \), investment will equal saving and the country will neither be a creditor or a debtor. When \( x > D \), then investment will exceed saving. There will be a short-term capital inflow and a positive
optimal ratio \( f = \text{debt/capital} > 0 \). The constraint that there be no default means that there is a maximal debt, \( f_{\text{max}} \), such that in the event of a bad state of nature \( b_2 = b' \), the level of consumption would exceed \( C_{\text{min}} \).

**Summary Short-debt model**

*Curve ABDEF is the constrained optimal ratio debt/capital,* in the short-term capital flow model. Expected utility is maximized along this curve. Insofar as the debt deviates from the curve, expected utility is reduced. The optimal debt is a benchmark of performance. *Debt crises result because the actual debt deviates from the optimal debt.* If the debt exceeds \( f_{\text{max}} \), due to non-optimal behavior of the public and private sectors, then with probability \( (1-p) > 0 \) consumption will be less than \( C_{\text{min}} \) and there will be a default. Alternatively, the likelihood of default rises continuously as the utility of consumption \( C_2 \) declines.

**Example: Emerging Markets**

In chapter seven, emerging market countries during the period 1979-2001 were divided into two groups: those that defaulted/renegotiated their debts with either official or private creditors\(^\text{23}\) and those that did not. The relation between the debt/GDP ratio and the expected net return did not accord with the optimal ratio in figures 3 in either set of countries examined (renegotiate/no-default). The main difference between the two groups concerned the excess debt, \( \omega = (f - f_{\text{max}}) \), the deviation between the actual debt/GDP ratio from the maximal debt/GDP ratio. The \( f_{\text{max}} \) is calculated in chapter seven, based upon the short-term stochastic optimal control model in chapter two.

When there is an excess debt - the debt ratio \( f_t \) above the line \( f_{\text{max}} \) in figure 3 - the economy is vulnerable. In the event of a bad shock, the level of consumption would fall below the minimum and would default. The bad shock will occur with probability \( (1-p) \). In the empirical applications of the short-term capital model to emerging markets, a *Warning Signal* \( \omega = (f - f_{\text{max}}) \) is equation (10). A *"flashing red" Warning Signal of a debt crisis occurs when* \( \omega > 0 \).

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\(^{23}\) Define default/renegotiation as a condition where the scheduled debt service is not paid on the due date under the original contracted conditions.
(10) \( \omega = f_i - f_{\text{max}} \)

Default is often a political decision, where international organizations and foreign countries are directly involved in bailouts and debt forgiveness. In the absence of bailouts, the excess debt is a sufficient condition for default. Our results based upon panel data were that:

In the cases where the debt was rescheduled/defaulted, the excess debt was positive in 84% of the cases. In the cases where there was no excess debt, default occurred in 17% of the cases.

A specific example concerns *Mexico and Tunisia*. Mexico defaulted to official and to private creditors during the period 1983-96, whereas Tunisia did not. Table 2 describes the relevant data from which one can infer that Mexico was more likely to default than Tunisia.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mexico - default</th>
<th>Tunisia - no default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt/GDP ( f )</td>
<td>0.45</td>
<td>0.61</td>
</tr>
<tr>
<td>Mean net return ( b-r )</td>
<td>0.057</td>
<td>0.107</td>
</tr>
<tr>
<td>Variance of net return ( \sigma^2 )</td>
<td>((.16)^2 )</td>
<td>((.09)^2 )</td>
</tr>
<tr>
<td>( z = (b-r)/\sigma^2 )</td>
<td>2.23</td>
<td>13.21</td>
</tr>
<tr>
<td>( \omega = f - (f_{\text{max}}) )</td>
<td>0.322</td>
<td>-0.116</td>
</tr>
</tbody>
</table>

Compare table 2 with the summary above. *First*: Tunisia, which did not default, had a higher debt ratio than did Mexico, which defaulted. *Second*: the risk adjusted net return \( z \) was very much higher in Tunisia. *Third*: the Mexican debt ratio was above the maximal debt \( f_{\text{max}} \) in figure 3. The excess debt, which leads to a probability of default in the bad case, is \( \omega \) in the last row. It is very large and positive. By contrast, Tunisia did not have an excess debt. The \( \omega \) for Tunisia was negative; the debt ratio was below the debt-max level.
Dynamic Programming Optimization in long-term debt models

For many countries the main obligations to foreigners arise from direct investment and portfolio long-term debt and equity investment. These forms of "debt" have no maturity date, but must be serviced regularly with interest and dividend payments. The modeling of optimal long-term debt in continuous time over an infinite horizon is very different from the modeling of short-term debt in discrete time described in above. In the long-term debt model, there is no maturity date. Bankruptcy can only occur if net worth, to be defined below, is negative. The optimal controls will prevent that from occurring, but the actual behavior may be non-optimal.

Consumption and the growth of the debt are described by equations (7) and (8) above. The two sources of uncertainty are the productivity of capital and the real interest rate, which may be correlated. First, the optimality criteria are discussed. Second, we describe two models with alternative stochastic processes concerning the sources of uncertainty: the Prototype Model and the Ergodic Mean Reversion Model. Third, we indicate why the literature that uses the "inter-temporal budget constraint" (IBC) is inadequate. Fourth, we explain why and how we use the Dynamic Programming analysis. Fifth, the conclusions concerning the optimal debt/net worth ratio, capital/net worth ratio and consumption/net worth are stated. Sixth, we give an example of the implications of the DP analysis by providing Early Warning Signal of Argentine crisis of 2000-2001. In chapter eight, we give an example of the interaction of an overvalued currency and excess debt in producing the Asian crises 1997-98.

Optimality Criteria

Several reasonable optimality criteria are used in the mathematical finance literature. Usually the criterion is to select the control/decision variables, consumption, debt or capital subject to constraints, to maximize the expectation (E) of the discounted value of a concave utility of consumption $U(C_t)$ over an horizon $(0,T)$, where $T$ may be infinite or finite. These are equations (11a) or (11b). In the infinite horizon case, the discount rate is $\delta \geq 0$. The expectation is taken over the stochastic variables: the productivity of capital and real rate of interest.
Analytic solutions of the dynamic programming equation can be obtained if the utility function is HARA\(^{24}\) described in equations (11a) and (11b). The coefficient of risk aversion is \((1-\gamma) > 0\). The lower is \(\gamma\), the greater is the risk aversion. Negative and zero values of \(\gamma\) imply considerable aversion to risk. In the case where \(\gamma \leq 0\), the utility of a zero consumption is minus infinity. When \(\gamma = 0\), the utility function is logarithmic.

\[(11a) \ J_1 = E \int_T^t C_t \gamma e^{-\delta t} \, dt \quad \gamma \neq 0 \quad T > t > 0 \]
\[(11b) \ J_2 = E \int_T^t \ln C_t e^{-\delta t} \, dt \quad \gamma = 0 \quad T > t > 0 \]

Two constraints are imposed. Consumption is always positive. Net worth must always be positive. Define net worth \(X_t > 0\) as "capital" less debt. A negative debt is a financial asset. Unless constraint \(X > 0\) is imposed, Ponzi schemes are possible: borrow to finance consumption and borrow more to service the debt. In that case, capital does not grow. As the debt continues to grow exponentially, net worth will be driven to negative values. The constraint that net worth is always positive precludes Ponzi schemes.

There are two subjective variables, the discount rate and risk aversion. The discount rate is just another way of specifying the length of the horizon. A high discount rate places the emphasis upon what occurs in the near future, and essentially disregards the far future. A discount rate \(\delta > 0\) is necessary to derive a finite optimum over an infinite horizon if \(\gamma \geq 0\), whereas if \(\gamma < 0\), then a discount rate is not necessary to derive a finite optimum over an infinite horizon.

Whenever the utility function is logarithmic, the optimal ratio of consumption/net worth equals the "discount rate". Consumption is social consumption, government plus private consumption expenditure. Low taxes and high government expenditures raise social consumption. Since the discount rate is arbitrary, this quantity can rationalize any social consumption policy. If populist policies lead to a high rate of social consumption/GDP, it can be "rationalized" as optimal policy with a high discount rate. Weight the present highly relative to the future. If the dictator, a Chairman Mao, follows policies that depress social consumption, it can be "rationalized" as optimal policy with a low discount rate. Weight the future highly relative to the present.

\(^{24}\) That is the reason that Merton used HARA. Otherwise, the DP equation must be solved numerically using a computer.
Criterion J₃ in equation (12) does not involve the arbitrary discount rate. Quantity J₃ is the expected growth rate of net worth over a horizon of length T, given any constant ratio c of consumption to net worth, Cₜ/Xₜ = c > 0. Since consumption is a constant fraction of net worth the maximization of J₃ is the same as the maximization of the growth rate of consumption from an arbitrary initial level.

\[
\begin{align*}
(12) \quad J_3 &= \frac{1}{T}E \left[ \ln \frac{X_T}{X} \mid \frac{C_t}{X_t} = c > 0 \right] \\
&= \frac{1}{T}E \left[ \ln \frac{C_t}{C} \mid \frac{C_t}{X_t} = c > 0 \right], \quad X = X(0), C = C(0)
\end{align*}
\]

Criteria J₁ and J₂ allow us to solve for both the optimum debt/net worth ratio, capital/net worth and the optimum consumption/net worth, whereas criterion J₃ only allows us to solve for the optimal debt/net worth and capital/net worth. We explain in chapter three that the same optimal ratios of debt/net worth and capital/net worth are obtained whether we use criterion J₂ or J₃.

There is another criterion, which reflects extreme aversion to risk. The consumption in any period depends upon both the controls/decision variables - consumption, capital or debt - and the stochastic productivity of capital. Suppose that there is a finite set of productivities of capital and a corresponding likelihood function. The max-min criterion of optimality is to select the controls that maximize the minimum expected values of consumption for very large values of risk aversion. Fleming (2005) analyzes this very conservative case.

**Stochastic Processes**

The sources of uncertainty are modeled as stochastic processes in continuous time. The prototype model assumes that both the productivity of capital bᵣ and the world real rate of interest rᵣ can be described by statistical functions such as Brownian motion²⁵ with drift. The mean return on investment is b, but there is no mean reversion. The change in the return to investment from one "period" to the next is purely random with a zero expectation, Brownian motion. Similarly, there is a world real rate of interest at which the country can borrow or lend. The mean is r, but there is no mean reversion. The

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²⁵ A Brownian motion process has independent increments that are normally distributed. The expectation is zero and the variance is directly related to the length of the period.
change in the real rate of interest from one "period" to the next is just the Brownian motion.

An alternative stochastic process is that the productivity of capital is still Brownian Motion with drift but that the world real interest rate, dependent upon a large vector of factors, is described by **Ergodic Mean Reversion**. This stochastic process is described by the *Ornstein-Uhlenbeck* equation, which states that the change in the real rate of interest from one "period" to the next is not completely random. One part is a reversion to the mean, and the second part is Brownian motion. The net result is that the real rate of interest is normally distributed and converges to a distribution whose mean is $r$ with a positive variance. Mathematically, it is easy to reverse which variable is described by Brownian motion with drift, and which is described by Ergodic Mean Reversion. The stochastic processes in the two models are summarized.

<table>
<thead>
<tr>
<th></th>
<th>Return on investment $b_t$</th>
<th>real interest rate $r_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototype model</td>
<td>Brownian motion with drift</td>
<td>Brownian motion with drift</td>
</tr>
<tr>
<td>Ergodic mean reversion</td>
<td>Brownian motion with drift</td>
<td><em>Ornstein-Uhlenbeck</em></td>
</tr>
</tbody>
</table>

In chapter three, we derive the optimal debt ratio and consumption ratio in the Prototype model. In chapter nine, where we evaluate the United States current account deficits and debt ratio, we explain how the equations for the optimum differ in the two cases: prototype model, ergodic mean reversion.

*Inter-temporal Optimization: Stochastic Optimal Control, Dynamic Programming*

The standard approach in the economics literature concerning inter-temporal optimization is to maximize the expectation of the discounted value of the utility of consumption subject to an "Inter-temporal Budget Constraint" **IBC**. The inter-temporal case is treated as the analogue of the standard deterministic case of consumer choice. In the timeless case, the consumer has a utility function over a vector of goods, leisure and services whose prices are given and the consumer has a fixed amount of resources, money and time. The constraint is that the choice is restricted to the amount of resources available. *The budget constraint is known with certainty* since prices and resources
available are known when the choice is made. The IBC is of an entirely different nature. The object of an "inter-temporal budget constraint" is to prevent a "free lunch", or engage in a Ponzi scheme where debts are never repaid. The IBC imposed is a terminal condition. At finite date $T > 0$ the debts are cleared, debt $L_T = 0$. We now explain why the IBC is inappropriate in a stochastic environment/a world of uncertainty. Instead, one must use the techniques of stochastic optimal control/dynamic programming.

From equation (7), the debt $L_T$ at time $T$ is the initial debt $L(0)$ plus the sum of the excess of expenditures for consumption $C_t$ plus investment $I_t$ plus interest on the debt $r_t L_t$ less Gross Domestic Product $Y_t$. The IBC is that the debt is paid off at the terminal date. The condition that $L_T = 0$ implies the IBC, the sum of absorption ($C_t + I_t$) is equal to the sum of the Gross National Product ($Y_t - r_t L_t$).

The stochastic variables (in bold letters) are $Y_t$ the real GDP and $r_t$ the real interest rate. Given the uncertainty concerning the productivity of capital and real interest rate, the future is unpredictable. At any time $s < T$ when the debt is $L_s$, how can anyone know if any country will/will not be, violating the IBC?

The IBC is unknowable, unenforceable and is a non-operational concept. If a country has a debt $L(0)$ at the present, how can one know if the IBC will be satisfied even if a given policy - a sequence of investment and consumption - is followed? The reason is that $Y_t$, $r_t$, the real GDP and interest rate are stochastic variables with Brownian Motion components. For example, when the price of oil (during the oil crises periods) was high the oil producing countries and the oil importing countries expected it to continue. In the former huge investment and consumption projects were undertaken in the expectation that the real GDP would remain high. In the oil consuming countries, costly energy saving policies were imposed. These anticipations did not materialize and the oil producing countries were saddled with large debts.

This profound deficiency of the IBC approach led Fleming and Stein\(^{26}\) to use dynamic programming DP approach\(^{27}\), which features prominently in this book.

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\(^{27}\) The seminal work was by Bellman The DP approach is generally used in the mathematical finance literature, starting from the work of Robert Merton.
The Dynamic Programming DP/Stochastic Optimal Control Approach SOC\textsuperscript{28}\textsuperscript{29}

Our underlying models are Markov diffusion processes where the future evolution of the system depends upon the present state and not at all upon the paths leading up to the present state. The system is stochastic, unpredictable. Even if one specified the controls/decisions\textsuperscript{29} from the present to any future date, the future is unpredictable because there are many paths that the system can take due to the stochastic processes describing the real GDP and the real interest rate. At each instant of time the "controller/decision maker" knows the state of the system, and only has information up to the present. Since, the controller cannot anticipate the future, the DP approach involves a multi-stage decision process. The principle of optimality of DP is that: whatever the initial state and the initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. In a stochastic system, the optimal controls selected at any time depend upon the current information available and enter as feedback functions of the currently observable state. This is very different from the IBC approach.

The state variable in the stochastic systems discussed in this book is net worth $X_t$ defined as "capital" $K_t$ less debt $L_t$. The change in capital is investment over the period, and the change in the debt is equation (7). The latter involves the stochastic variables, the productivity of capital and the real rate of interest.

The DP solutions of the optimization of the expected discounted value of utility ($J_1$, $J_2$ in equations 11a, 11b), given the stochastic processes, involve the Hamilton-Jacobi-Bellman (HJB) equation discussed in chapter three. The DP analysis of inter-temporal optimization is quite technical, however the optimal debt/net worth $f$ in the HJB equation can be explained in terms of the well known Tobin Mean-Variance M-V model.

The optimal debt/net worth $f_t$ in the HJB equation is chosen to maximize component $W$, equation (13).

(13) $W = max_f [\text{Mean} - (\text{risk aversion}) \text{Risk}].$

\textsuperscript{28} This section is an intuitive discussion of chapter three, which is based upon techniques used in the mathematical finance literature.

\textsuperscript{29} Controls and decisions are used interchangeably.
The **Mean** term \( M(f_t, c_t) \) is a *linear* function of the debt ratio and the consumption ratio. It is the percentage change in net worth \( dX/X \) if there were no uncertainty. **Risk** is \( R(f_t) \) and \((1-\gamma)\) is risk aversion. The risk term concerns the variance of the percent change in net worth. In the logarithmic cases \((\gamma = 0)\), risk aversion is unity. Term \( R(f_t) \) contains the variances of the productivity of capital, the interest rate and their correlation. Stochastic term \( R(f_t) \) is a *quadratic* function of the debt ratio. A unique optimal ratio of debt/net worth is derived that maximizes \( J_1 \) or \( J_2 \) in equations (11a) and (11b). This maximization involves the maximization of \( W \), which can be interpreted as M-V expected utility. A graphical explanation of equation (13) presented in chapter three provides the intuition behind the DP results.

**Implications of the DP solution of the long-term debt model**

The Stochastic Optimal Control/Dynamic Programming analysis is used to derive the inter-temporal optimal conditions. The debt/net worth ratio \( f_t = L_t/X_t = f^* \) that maximizes performance criterion \( J_1 \), equation (11a) is the one that maximizes equation \( W \) in (13). The derived optimal debt in equation (14) is a benchmark measure of performance in a stochastic environment. Net worth is capital less debt. Therefore, the optimal ratio \( k^* \) of capital/net worth is \( k^* = f^* + 1 \). In the logarithmic case \( J_2 \) equation (11b) where risk aversion \((1-\gamma) = 1\), the *optimum debt/net worth* is:

\[
(14) \quad f^* = \frac{(b-r)}{\sigma^2} + f(0), \quad \sigma^2 = \text{var} (b_t - r_t).
\]

Several crucial variables are in this equation. First, variable \( b \) is the *mean* productivity of capital or return on investment, \( r \) is *mean* real interest rate. In the logarithmic cases \( J_2 \) and \( J_3 \) risk aversion \((1-\gamma) = 1\). Variable \( \sigma^2 \) is the variance of the quantity \((b_t - r_t)\), the current productivity of capital less the current interest rate, so that it also contains a covariance term. The intercept \( f(0) \) is the optimal ratio of debt/net worth that minimizes risk. When the correlation coefficient between the growth rate and interest rate is less than the ratio of the standard deviation of the productivity of capital/standard deviation of the interest rate, the intercept \( f(0) \) is negative. The optimum debt ratio \( f^* \) in the Prototype model is equation (14), which is our benchmark of performance. Equation

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30 This is \( dX/X \) but it is not the growth rate, which is the percentage change per unit of time.
(14) is graphed in Figure 4 as line $U-S$. The debt ratio $f = L/X$ is plotted on the ordinate and the risk adjusted mean rate of return $z = (b-r)/\sigma^2$ is plotted on the abscissa.

Figure 4. Optimal Ratio Debt/Net Worth $f^*$ is line $U-S$, when risk aversion is unity. In the Prototype model, the risk adjusted net return $z = (b-r)/\sigma^2$, $\sigma^2 = \text{var} (b_t - r_t)$. Optimal capital/net worth $k^* = 1 + f^*$.

**Summary and implications of the DP analysis**

Consider two countries, which differ greatly in terms of wealth and income. There is no necessary relation between per capita wealth and the risk adjusted mean net return $z = (b-r)/\sigma^2$. In rich/developed country I, the risk adjusted mean net return $z = z_1$ and in poor/emerging market country II the risk adjusted mean net return $z = z_2$. In the situation described in figure 4, it is optimal that the poor country should be a creditor of the rich country because the mean return per unit of risk $z = (b-r)/\sigma^2$ is higher in the rich country.

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31 These propositions refer to the case where risk aversion $(1-\gamma)$ is unity, the logarithmic case.
Either the mean net return \((b-r)\) is higher or the risk \(\sigma^2\) is lower in rich country I than in poor country II.

The expected growth of net worth and consumption is maximal, for any consumption ratio, when the debt/net worth ratio\(^{32}\) is optimal at \(f^*\). As the debt ratio rises above the line U-S which describes the optimum \(f^*\), the expected growth rate declines, and the risk - the variance of the growth rate - increases.

Objective measures of vulnerability to external shocks are implied by the analysis. Vulnerability is taken as a situation where, if the debt is to be serviced, consumption must be reduced when there are external shocks. Say that consumption is a constant proportion of net worth. This proportion may or may not be "optimal". As the actual debt ratio rises above the optimum, say because non-optimal policies are undertaken by the public sector, the expected growth rates of net worth and consumption decline and their variances rise. The probability of a decline in consumption rises and the probability of a debt crisis increases continuously as the debt ratio exceeds the optimum. Therefore, the vulnerability to shocks increases continuously as \(f\) rises relative to \(f^*\).

The level of the ratio\(^{33}\) of debt/GDP per se is not a relevant variable in producing a crisis. Instead it is the excess of the actual debt ratio over the optimal \(f^*\) that raises the probability of a crisis. A Warning Signal \(\Psi_t = (f_t - f^*)\), based upon available information, is that the debt ratio is rising above line U-S in figure 4. When the mean net return per unit of risk \(z = (b-r)/\sigma^2\) is falling, the optimal debt ratio should be declining. If, however, the debt/GDP ratio is rising because non-optimal policies are followed, it is more probable that the debt ratio lies in the region above the curve \(U-S\).

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\(^{32}\) The ratio of capital/net worth is equal to 1 plus the debt ratio.

\(^{33}\) The ratio of \(h = \text{debt/GDP}\) is positively related to the ratio \(f^* = \text{debt/net worth}\). The equation is:  
\[ h = (1/b)f/(1+f), \]  
where \(b\) is the mean productivity of investment. Therefore, one can speak of the ratios \(f\) and \(h\) interchangeably.
Example: Argentina: From Triumph to Defaults

Severe crises result from an interaction between an excess debt and a misaligned exchange rate. The interaction between them is the subject of the Asian crises chapter eight. In the present section, we give an example of how we apply the SOC/DP analysis of excessive debt to Argentina that went from Triumph in the early 1990's to Tragedy in 2001. Warning Signals are based upon estimates of excess debt $\Psi_t = f_t - f^*_t$ using available information.

Michael Mussa's (2002) retrospective description, and the International Monetary Fund's Independent Evaluation Office Report (2004), of the Argentine crisis can be integrated with the SOC/DP analysis above. A new economic policy - the convertibility plan - was instituted in the spring of 1991 to deal with the hyperinflation that existed at the beginnings of the 1990s. The currency was pegged to the $US and a currency board arrangement limited domestic money creation. This plan was successful. During the period 1993-98, the inflation rate was below 3% and the growth rate was about 4% per annum. Whereas most of the miracle Asian economies collapsed into crisis from mid 1997 to early 1998, Argentina became the darling of the world credit market. It was able to float large issues of medium to long-term debt on the world credit markets at comparatively modest spreads over the US Treasuries.

Not only did the world credit markets and the International Monetary Fund applaud the Argentine policies, but also several academic economists viewed Argentina as a model of growth. Dornbusch (Lecture II: 1998) wrote: "A currency Board arrangement, a fixed exchange rate and a central bank that has no discretionary power over the money supply…is a very good system…One spectacularly successful case…is Argentina…the Argentine experience is the one that deserves most attention because, one, it has lasted, and two, it has been extremely successful as a cornerstone of reform in an economy, and three, it has produced an average growth of six per cent".

Barely a few years later in 2001, Argentina's decade long experiment ended in tragedy. The banking system was effectively closed at the beginning of December 2001, the exchange peg was gone, the peso was trading at substantially depreciated rate against the $US. Argentina defaulted on its sovereign debt and was transformed within barely

\[34\] The description of the Argentine experience in this section often paraphrases Mussa's monograph.
two years from the darling of the emerging market finance to "the world's leading deadbeat".

The reason why the financial markets, the International Monetary Fund and academic economists failed to anticipate the crisis was that their attention was focused upon the monetary sphere - since inflation is a monetary phenomenon - and not upon the external debt. The debt did not alarm them because the debt did not seem to be high relative to that prevailing in many industrial countries. A benchmark of an excessive debt was lacking. Our analysis implies that debt crises are not produced by the level of the debt/GDP but by the excess of the actual debt over the optimum debt ratio.

The fundamental causes of the disaster were the growth in social (public plus private) consumption and a fixed nominal exchange rate pegged to the $US. The various levels of the Argentine governments succumbed to political pressures to spend significantly more than was raised by taxes. Much of the fiscal problems arose because the provinces retained the initiative for public spending, but the central government was ultimately responsible for raising revenues and servicing the debt. Since Argentina was thought of so highly by the financial markets, it was able to finance the excess spending by borrowing US dollars in the international markets at favorable interest rates.

An excess debt means that the debt ratio rises above the curve U-S in figure 4. In the Prototype model, the optimal debt ratio is equation (14). A relatively general way of taking all of these factors into account is to graph the normalized variables. The normalized return to investment $b^*_i = (b_i - E(b))/\sigma_b$ labeled B1_AR in figure 5. It is the deviation of the return from its longer-term mean per unit of risk. The external $US denominated debt/GDP, labeled DBTGDPR_AR in figure 5, is also normalized. It is equal to the (debt/GDP - mean)/standard deviation. The debt ratio rose by two standard deviations from 1992-2001. However, the return on investment $b_t$ was declining from 1997-2001.

The rise in the actual debt ratio and the decline in $b_t$ the return/risk in figure 5 corresponds to a rise in the debt ratio above the curve U-S for the optimal debt ratio in

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35 In general, in the equation for optimal debt the variance is multiplied by risk aversion. In the logarithmic case it is unity, as in equation (14). There is also a constant $f(0)$ in equation (14). By using normalized variables, we are evaluating excess debt relative to its average value over a long period, where risk aversion and $f(0)$ are given.
figure 4. An "excess debt" is generated. Insofar as there is an excess debt, the expected growth rate of GDP declines and its variance rises. The variance comes from the external shocks, which are disturbances to the productivity of capital (GDP/capital), the real rate of interest, and their correlation. Because of the non-optimal government policies, the Argentine economy became more vulnerable to shocks of the net return from its longer-term mean.

The major shock was the collapse of Brazil's crawling peg early in 1999, which led to a negative shock to the Argentine productivity of capital. When Argentina was forced to depreciate its currency - abandon the peg - the real rate of interest was positively shocked, because the debt was denominated in $US. Consumption would have to be reduced, if Argentina was to service her debts. Confronted with this choice, Argentina defaulted.

Our conclusion is that there was an Early Warning Signal (EWS) of a sustained excessive debt, based upon available information. The debt ratio per se is not an EWS, whereas the excess debt $\Psi = f - f^*$ is an EWS of a debt problem/crisis/default/renegotiation. Theoretically this is the movement of the debt ratio above the curve US, and empirically it is that $b^*_t = (b_t - \text{mean})/\text{st. dev}$ is falling significantly but the debt/GDP ratio is rising significantly for a period of years. This approach allows for gradual structural change in the productivity of capital.
ARGENTINA

DBTGD = debt/GDP, B1 = b = return on investment
normalized variables: (variable - mean)/st. dev.

Figure 5. Debt/GDP (DBTGD_AR) normalized, return on investment per unit of risk
(B1_AR) = (b, - mean)/st. dev.
Conclusion

The contribution of this book has been to answer the following questions.

- What is a theoretically based empirical measure of a "misaligned" exchange rate that increases the probability of a significant depreciation or a currency crisis?
- What is a theoretically based empirical measure of an "excess" debt that increases the probability of a debt crisis?
- What is the interaction between an excess debt and a misaligned exchange rate?

In this Overview chapter we sketched the theoretical framework and gave several examples of how the theoretical analysis is applied. This book reflects interdisciplinary work that has been undertaken with co-authors. The NATREX model was developed and applied with co-authors who are economists. The use of stochastic optimal control and dynamic programming to explain optimal debt and endogenous growth was developed with co-authors who are mathematicians.

Chapters 2-4 are theoretical and chapters 5-9 are applications of the theoretical material. A detailed application of stochastic optimal control/dynamic programming for long-term debt in continuous time is in chapter nine concerning the United States current account deficits. The Argentine example above just gives a flavor of the approach. The short-term debt model is applied to the emerging market countries in chapter seven. The interaction between misaligned exchange rates and excess debt is applied to the East Asian crises. The application of the NATREX model to the Euro-U.S. dollar exchange rate is chapter five, and to the real exchange rate of the transition economies in chapter six.

Now that the reader has had a guided tour of some of the highlights, he can read the rest of the book either systematically or selectively by choosing the chapter that is closest to his interests.
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