The failure of Woodford's model of the channel system in the cashless economy

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Abstract:

Woodford claims to present a cashless and frictionless model of the channel system of interest rate control. This note explains why Woodford's frictionless model omits an essential feature of channel systems: the need for clearing e-cash balances at the central bank. Without the existence of some form of final settlement on the books of the central bank channel systems collapse. Woodford makes this conceptual error because he assumes that moneyless accounting systems of exchange can be applied to model e-cash economies.

JEL classification**:** B 40, E40, E42, E50

Key words: Cashless, frictionless, e-cash, accounting systems of exchange, channel system.

Introduction

Woodford (2003) claims that his analysis of monetary policy in a cashless or frictionless model captures the properties of the channel system of interest rate control employed by central banks. The channel system is Woodford's name for the electronic Real Time Gross Settlement systems employed in New Zealand, Australia and a growing list of other countries. These systems have effectively introduced the world of electronic cash (e-cash) to all high value transactions that are cleared through central banks. It is the objective of this note to explain why Woodford's cashless or frictionless model omits an essential feature of this channel system of interest rate targeting in e-cash economies.

In such systems, it is necessary for all the banks and financial institutions to clear e-payments through the central bank if the channel system is to work effectively.[1]

This step is essential as it gives the central bank the power, as the monopoly supplier of clearing e-cash balances, to impose losses on clearing banks by imposing penalty rates on the repurchase arrangements that apply to the daily dealings between the central banks and clearing banks under channel systems.

Thus for the channel system to operate effectively all clearing banks must hold e-cash balances at the central bank – in Australia they are called 'exchange settlement funds' that are held in 'exchange settlement accounts'. From this perspective Woodford (2003, p. 32) makes a mistake when he claims that it is possible for central banks to determine the policy rate of interest in the channel system even at the cashless limit, *without the need for clearing balances at the central bank*. As a description of e-cash channel systems this claim is simply false.

II The mechanics of the channel system - real time gross settlements in e-cash systems

The operation of the channel or settlement system in Australian was clearly explained by Campbell (1997) who highlights the following features of the system that are relevant to this note.

First, the cash rate is determined by the demand for and the supply of e-cash in a 'market' where the central bank is the monopoly supplier of e-cash. The demand for e-cash, or what the Reserve Bank of Australia calls 'exchange settlement funds', comes from the commercial banks who hold settlement accounts at the Reserve Bank. The supply of e-cash is determined by the Reserve Bank in both its open market operations (now mostly repurchase agreements) to influence the system-wide supply of e-cash, and by intra-day or overnight repurchase trading, to ensure that the supply of e-cash matches the demand for e-cash at the Reserve Bank's policy or cash rate. In this system, payments from the banks to the Reserve Bank reduce the aggregate supply of settlement funds or e-cash; and payments from the Reserve Bank to the banks increases the aggregate supply of settlement funds or e-cash. In this process the Reserve Bank pays no particular attention to monetary or credit aggregates but aims to ensure that the 'market' rate matches the policy rate.

The level of settlement funds held by profit maximising commercial banks at the central bank is a function of the opportunity cost measured by the net interest income foregone on settlement balances, and any cost penalties that the central bank may impose if clearing institutions are persistently in deficit with it. To minimize those costs, commercial banks must manage their daily e-cash flow and

the central bank must manage the aggregate daily money market e-cash position to enforce its policy rate. In effect the Reserve Bank as the monopoly supplier of settlement funds or e-cash is acting as a price setter but one that is acting in the public interest by setting interest rates to achieve macroeconomic objectives.

Second, the channel element of the system consists of two rates that are posted 25 basis points either side of the policy or cash rate and which move with the policy rate when it is changed. The function of these two rates is as follows. The lower bound to the channel, the cash rate minus 25 basis points, is the rate of interest paid on surplus settlement balances held by commercial banks at the Reserve Bank. The rate was set at this level to discourage the commercial banks from holding excess settlement funds at the Reserve Bank. As Campbell explains:

> "…the Reserve Bank agreed to pay interest on banks' ES balances. At first this rate of interest was set at 10 basis points (0.1 of a percentage point) below the cash rate target. This margin proved attractive for banks, which were happier to hold ES funds at this rate than to lend cash in the market to other institutions of inferior credit standing than the Reserve bank. Accordingly, balances in ES accounts rose sharply…. This unwillingness to lend cash had the effect of making it more difficult to achieve the cash rate target."(Campbell 1997, p.8)

Consequently the Reserve Bank reduced the interest paid on settlement balances to 25 basis points below the cash rate and this had the desired effect of reducing the settlement balances held at the Reserve Bank.

The function of the lower bound to the channel is thus to keep the settlement funds at the Reserve Bank at a level consistent only with the planned clearing requirements of the commercial banks. With settlement balances at this level the Reserve Bank's influence over the daily cash rate is improved because any

system-wide shortage of e-cash can only be met from the Reserve Bank and not from the draw-down of excess settlement accounts or e-cash balances.

The upper bound of the channel, the cash rate plus 25 basis points is the penalty rate that the Reserve Bank can impose if clearing banks run short of settlement balances and are forced (automatically) to engage in intra-day or overnight repurchase agreements. Commercial banks therefore have an incentive to manage their daily use of settlement balances at the Reserve Bank so as to avoid any imposition of the penalty rate.

The essence of the channel system is therefore that it provides a practical mechanism through which the Reserve Bank can impose its cash rate on the money markets but without causing a disruption to the payments system while at the same time reducing the settlement (Herstatt) risk that can occur in large value deferred net-settlement systems.

The system works effectively in the sense that the cash rate is under the control of the central bank so long as it remains the monopoly supplier of e-cash and the money market is dependent on the central bank for the daily supply of system-wide e-cash. The extent to which the banks are dependent on the central bank for daily e-cash is therefore important in accounting for the degree of convergence between the policy or cash rate and the 'market' rate.

The two key operational points to take from this description of the channel system as it operates in Australia are as follows:

i) Although it is an electronic payments system and physical cash and cheques have disappeared, it is an e-cash system that rests on the

existence of electronic bookkeeping entries to transfer and track payments between commercial banks. The form of money has evolved from paper to e-cash but money (= cash) has not vanished.

ii) Consequently it is essential that commercial or clearing banks hold settlement funds or e-cash balances at the central bank. Without the existence of such clearing balances the channel system would collapse because the central bank would have no means of imposing its policy rate on the money market.

The relationship between the Australian settlement system and the US system now that the Federal Reserve pays interest on reserves and allows more open access to the discount window has been described by Keister, Martin and McAndrews (2008). Their analysis makes it clear that the difference between the two systems is one of degree not kind. In particular, the Federal Reserves' current payment of interest on excess reserves at the cash rate creates a variant of the channel system that Keister *et al.* describe as a 'floor system' because the lower bound of the channel coincides with the cash rate.

III **Woodford's cashless economy as a moneyless system**

Woodford (2003) accurately describes many features of the channel system outlined above but his analysis breaks down when he attempts to equate the properties of such e-cash systems to what he calls the cashless or frictionless model.

The fundamental problem with Woodford's hypothetical cashless or frictionless model is that it is a moneyless system. This is a fundamental issue that has

bedevilled monetary theory for decades and there is no space to discuss it in detail here. Nevertheless, it is important to be aware that there is long and rich literature dealing with the limitations of attempts to impose a role for money onto Walrasian or Arrow-Debreu general equilibrium systems where there is no role for money. Such general equilibrium systems have no role for money because the auction necessary for their construction reduces them to models of perfect or costless barter. Hahn (1965) was the first to make this point formally but McCallum (1985) also correctly identified the models proposed by Black (1970) or Fama (1980) as accounting systems of exchange that had no role for money. Other important contributions were made by White (1984) and Hoover (1988a, 1988b) who exposed the futility of reducing money to only its unit of account or *numeraire* function in a perfect barter model and explained why the means of final settlement was an indispensable property of any monetary economy.

Unfortunately, Woodford (2003) has followed Black and Fama by employing a moneyless or perfect barter general equilibrium system as the theoretical basis for his analysis of the channel system. [2] Such models have no essential role for e-cash and so no role for a central bank or the channel system outlined in section II above. As a consequence of conflating cashless with e-cash systems Woodford makes false claims about the theoretical foundations of monetary policy on the basis of his model of the cashless economy.

Conflation of cashless accounting systems of exchange and e-cash channel systems

The conflation of accounting systems of exchange or the Arrow-Debreu model of complete markets with the channel system is evident from Woodford's description of the relationship between his model and the channel system:

"In the discussion of interest rate determination under the present-day channel system, I have supposed that there is a demand for at least a small quantity of central-bank balances for clearing purposes, and these are held despite the existence of a small opportunity cost (25 basis points on average). But once the idea has been accepted that the central bank can vary the overnight interest rate without ever having to vary the size of this return spread, *the function of the system no longer depends on the existence of a clearing demand*". (Woodford 2003, p. 32 emphasis added)[3]

It should now be apparent that in progressing from the first to the second sentence in this paragraph Woodford makes a fundamental conceptual mistake. As explained above, and Woodford outlines it himself in the first sentence, a *necessary feature* of the channel system is the existence of clearing balances held by the banks at the central bank. Given the legal constraint it is the threat of losses and increased costs that the central bank may impose on clearing banks who misuse its repurchase facility that induces clearing banks to manage their clearing balances at the central bank. Without those clearing balances the central bank would lose control over interest rates. *It is therefore not possible to implement a channel system in an economy that has no clearing demand at the central bank.*

Woodford's claim nevertheless follows logically from the frictionless and cashless properties of his model as an accounting system of exchange when he explains that there is no need for a means of final settlement in his cashless economy:

"Under the assumption of frictionless financial markets it is natural to suppose that *no "monetary" assets are needed to facilitate transactions*." (Woodford 2003, p. 62, emphasis added)

Woodford argues that money overcomes trading frictions so when a frictionless model is employed there are no 'monetary frictions' for money to overcome and money (cash) is redundant. The use of a frictionless model is therefore the source of the flaw in Woodford's modelling strategy because frictionless models are moneyless accounting systems of exchange while e-cash economies are monetary economies where all the frictions that money was invented to reduce still exist.

Woodford's description of his frictionless model clearly indicates that it is a moneyless accounting system of exchange which, it turns out, is isomorphic with the complete markets Arrow-Debreu model. In other words, in Woodford's cashless model there is no need for a means of settlement at all, whether in the form of cash or e-cash.[4]

In this context, the next question that naturally arises is whether the e-revolution and the evolution of e-cash means that the world is converging on the properties of the accounting system of exchange (a pure exchange economy) as conjectured by King (1999). Woodford (2003, p. 31-37) refers to King (1999) and Fisher Black (1970), who made a similar suggestion, to support the argument that fully 'deregulated' financial markets behave like a moneyless accounting system of exchange.

There is some ambiguity on this point in the literature because two issues have been conflated. First, there is the question of whether the evolution of money from paper to e-cash in 'deregulated' financial markets means that the economy

is converging on the properties of a cashless accounting system of exchange or the cashless Arrow-Debreu model of pure exchange. Second there is the question of whether the evolution of e-cash means that clearing will no longer take place through the central bank. Both of these questions should be answered in the negative.

First, as explained above, e-cash economies are not frictionless cashless economies. In e-cash economies all the frictions that money was invented to reduce still exist – they are not frictionless systems no matter how 'fast' the computer speed becomes. All financial transactions still occur in real time and time itself is friction that cannot be eliminated. As will be illustrated below, the frictionless and cashless model to which Woodford appeals is nothing more than an accounting system of exchange or an Arrow-Debreu world that rests on a time-0 auction. Such models have no role for e-cash or any money that acts as a means of final settlement.[5] As Woodford's model of the cashless economy belongs in the latter class of models it is not a suitable model of the e-cash channel system. The latter is a monetary economy - Woodford's frictionless and cashless economy is not.

Second, it is now apparent that the e-revolution has indeed improved the efficiency and reduced the settlement risk in high value transactions systems by enabling the implementation of settlement systems in real time. Furthermore, this state of affairs is likely to persist so long as fiat money systems exist and there is a need for central banks to manage such systems in the public interest. There is now little disagreement about this point. In such systems the e-revolution has actually improved the ability of central banks to control their

policy interest rates. This does not mean, of course, that other factors such as the globalization of financial markets may have weakened their ability along other dimensions.

To sum up, Woodford's (2003, p. 62) theoretical model of the cashless or frictionless world does not capture an essential feature of the e-cash channel system that he proposes to analyse because at the cashless limit the model has no role for e-cash as a means of final settlement. The absence of this crucial element from Woodford's model can be seen by examining the properties of his model.

Woodford's cashless economy as a moneyless accounting system of exchange

The model employed by Woodford (2003, p. 64) is described as an asset-pricing model with nominal assets. However it immediately becomes clear that the model need not contain cash and nominal assets in the cashless or frictionless state. Nominal assets exist and cash is necessary only when 'monetary frictions' exist. In its frictionless state there are no frictions for money to overcome rendering nominal magnitudes and cash (money) redundant. This explains why clearing balances at the central bank simply vanish or are rendered redundant at the cashless limit in Woodford's model. But without e-cash and the demand for clearing balances, the central bank cannot employ the channel system to set its policy rate in the cashless state of Woodford's model. To make this clear consider the following key equations and their properties in Woodford's frictionless and cashless model.

Woodford considers an economy made up of identical households who maximize the expected discounted sum of period utility of the form (equation numbers follow Woodford (2003, pp. 64-70):

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t u_t(C_t,\xi_t)\right\} \qquad (1.1)$$

The discount factor is $0<\beta<1$ and C_t is the consumption level of the economy's single consumption good. The variable ξ_t represents a stochastic disturbance to the period utility function.

Woodford then introduces a series of assumptions that effectively maps the properties of the cashless state of the model to the properties of the Arrow-Debreu complete markets model of pure exchange. In particular, he explicitly assumes complete markets, notes that he need not distinguish between particular assets under that assumption and explains that the infinite sequence of flow budget constraints is equivalent to a single lifetime intertemporal budget constraint. [6]

These assumptions ensure that looking forward from any date t this intertemporal budget constraint is written as:

$$\sum_{s=t}^{\infty} E_t Q_{t,s}[P_s C_s + \Delta_s M_s] \leq W_t + \sum_{s=t}^{\infty} E_t Q_{t,s}[P_s Y_s - T_s] \qquad (1.13)$$

Where the variables are defined as follows: $Q_{t,s}$ is a stochastic discount factor; E_t refers to the expectation conditional on the state of the world at date *t*; M_t is the end of period holding of cash or money; P_t is the money price of the consumption good; W_t is the beginning of period value of financial wealth; Y_t is an exogenous endowment of the single good; and T_t is net nominal tax collected by the

government. The symbol, $\Delta_s \equiv (i_t - i_t^m)/(1+i_t)$, where i_t^m is the nominal rate of interest paid on cash balances.[7] The budget constraint (1.13) must hold at each date and household wealth accumulation must satisfy a transversality condition:

$$\lim_{T\to\infty} E_t[Q_{t,T}\, W_T] = 0 \qquad (1.18)$$

This statement of the model applies to the case where monetary frictions supposedly exist because it includes *M* and nominal magnitudes, including the nominal rate of interest, i_t^m in Δ_s. The difficulties for Woodford (2003, p. 69) in applying the cashless version of this model as a theoretical foundation for the channel system are revealed when he suggests that "… since in the cashless economy there is no pecuniary benefit to holding money balances, household optimization requires that:

Either $M_t = 0$ (1.14)

or $i_t = i_t^m$ (1.15)

at each date and in each possible state (although which conditions obtains may differ across dates and across states)." But both expressions (1.14) and (1.15) are inconsistent with e-cash channel systems.

First, when expression (1.14) applies Woodford's model has been rendered truly cashless as $M_t = 0$, and it then satisfies the properties of McCallum's (1985) moneyless accounting system of exchange or the cashless Arrow-Debreu complete markets model of perfect barter. This means that nominal magnitudes such as i_t^m are not defined, there is no money or nominal price of the consumption good, C_t and obviously no clearing balances are required. From the perspective of the channel system e-cash has also vanished.

Second, in the case where (1.15) holds, the interpretation of the model is more convoluted. Clearly if expression (1.15) holds then positive amounts of money or cash balances may exist but they play no operational part in the model. This can be seen from inspection of expression (1.13) where $\Delta_s \equiv (i_t - i_t^m)/(1+i_t) = 0$ when condition (1.15) applies. In this case although positive money balances exist they play no role in the budget constraints of households, banks or the central bank. The interpretation of Woodford's model relevant to this state was accurately captured by Green's description of the model:

"In the cashless economy that Woodford models, the central bank can set a nominal interest rate directly by standing ready either to lend or borrow unboundedly at its policy interest rate (analogous to the Federal funds rate in the United States). However, *the outside money that is borrowed or lent is a redundant asset* in a complete system of markets for dated, state contingent consumption, and *plays no role in facilitating transactions*." (Green, 2005, p. 124)

This description applies to Woodford's model when expression (1.15) holds and the model is strictly speaking not cashless because in this state $M_t > 0$. However, as $\Delta_s = 0$ in expression (1.13) the cash that exists is nevertheless a *redundant asset* that plays no part in household budget constraints or in facilitating transactions.[8] In this frictionless state of the model there are no frictions that money can overcome so it has no role. By contrast, in e-cash economies those frictions still exist so such economies are not frictionless.

Consequently, on the face of it this modelling strategy under expression (1.15) looks suspiciously like a sleight of hand that raises more questions than it answers. In particular, if money is a redundant asset why is it included in the model? Why would agents hold it if it is not required to make transactions or is a redundant component of wealth? The obvious answer is that it should be dropped from the model because in a model with a complete system of markets

for state dated consumption under a time-0 auction, M_t has no function. The core of Woodford's frictionless model is invariant to the presence or absence of money as is evidenced by expression (1.14). In such a world of frictionless markets as is imagined to exist in an accounting system of exchange all commodities are equally 'money'. So, as Woodford (2003, p. 62) inevitably explains, under the assumption of frictionless markets it is then natural to suppose that *no monetary assets are needed to facilitate transactions*. Unfortunately in the world of e-cash channel systems central bankers cannot afford the luxury of that assumption.

IV **Concluding remarks**

The world of e-cash channel systems depend on the existence of e-cash as a means of final settlement. Money or cash in the form of electronic bookkeeping entries or e-cash must exist and banks must hold clearing e-cash balances at the central bank if it is to impose its policy interest rate as required under the channel system.

Consequently, Woodford's theoretical model of the e-cash channel system fails at the cashless limit because his core model is nothing more than a cashless accounting system of exchange. In such models e-cash and the channel system have no role.

The reason for Woodford's failure to provide a satisfactory theoretical foundation for the channel system in terms of a cashless or frictionless model can be attributed to a simple conceptual oversight that has dogged monetary theory for decades. Many monetary theorists have, for the last three decades at

least, been labouring under the belief that the moneyless accounting systems of exchange as personified by the Arrow-Debreu model of perfect barter or complete markets is the developed part of economic theory and monetary theory is by comparison underdeveloped. But this faith is misplaced.

The conceptual flaw in this approach to monetary theory was comprehensively documented in the literature in the 1980s but has subsequently slipped under the radar. This literature highlights the redundant role for money in perfect or costless barter general equilibrium systems, and stress the indispensable role for a means of final settlement, even if in electronic form, in any functioning monetary economy.

Woodford's implicit belief that the Arrow-Debreu or Walrasian general equilibrium system is the only relevant economic theory has led to his flawed attempt to model a computerised banking system. A model of costless or perfect barter (an accounting system of exchange) cannot provide the theoretical foundations for the e-cash channel system (or any of its variants). The two worlds have nothing in common. Electronic money is indispensable to modern settlement systems but is an inessential or redundant addition to Woodford's model. It seems that in his desire to avoid a role for monetary aggregates Woodford has gone too far and excluded entirely any means of final settlement, including e-cash, from his frictionless model.

LITERATURE CITED

Black, Fisher. (1970) "Banking and interest rates in a world without money." *Journal of Bank Research*, Autumn, 9-20.

Campbell, Frank. (1997) "The Implementation of Monetary Policy: Domestic Market operations." *Reserve Bank of Australia*.

Cochrane, John H. (2005) "Money as Stock." *Journal of Monetary Economics*, 52:3, 501-528.

Fama, Eugene F. (1980) "Banking in the theory of finance." *Journal of Monetary Economics*, 6:1, 39-57.

Friedman, Benjamin M. (2000) "Decoupling at the Margin: The Threat to Monetary Policy from the Electronic Revolution in Banking." *International Finance*, 3:2, 261-272.

Goodhart, Charles A. E. (2000) "Can Central Banking Survive the IT Revolution?" *International Finance*, 3:2, 189-209.

Goodhart, Charles A. E. (2009) "Liquidity Management." Symposium on Financial Stability and Macroeconomic Policy, *Federal Reserve Bank of Kansas City*, Jackson Hole, August 21, 2009.

Green, Edward J. (2005) "A Review of Interest and Prices: Foundations of a Theory of Monetary Policy." *Journal of Economic Literature*, XLIII, 1, 121-134.

Hahn, Frank H. (1965) "On some problems of proving the existence of equilibrium in a monetary economy." in *The Theory of Interest Rates*, pp. 126-135, edited by Frank. H Hahn and F. P. R. Brechling. London: Macmillan.

Hahn, Frank H. (1982) *Money and Inflation*. Oxford: Basil Blackwell.

Hoover, Kevin D. (1998a) "Money, Prices and Finance in the New Monetary Economics." *Oxford Economic Papers*, 40, 150-167.

Hoover, Kevin D. (1988b). *The New Classical Macroeconomics*: *A sceptical inquiry*. Oxford: Basil Blackwell.

Keister, Todd, Andrew Martin, and James McAndrews. (2008). "Divorcing Money from Monetary Policy." *Federal Reserve Bank of New York, Policy Review*, September, 41-56.

King, Mervyn. (1999) "Challenges for Monetary Policy: New and Old", in New Challenges for Monetary Policy, *Federal Reserve Bank of Kansas City*, 11-57.

Ljungqvist, Lars, and Thomas J. Sargent. (2004). *Recursive macroeconomic theory*, 2nd ed. Cambridge Mass: The M.I.T. Press.

McCallum, Bennett T. (1985) "Bank deregulation, accounting systems of exchange, and the unit of account: a critical review." In *The 'New Monetary Economics', Fiscal Issues and Unemployment*, edited by Karl Brunner and Allan Meltzer, Carnegie-Rochester Conference Series on Public Policy, 23, 13-46.

Rogers, Colin. (2006) "Doing without money: a critical assessment of Woodford's analysis." *Cambridge Journal of Economics*, 30, 293-306.

Wallace, Neil. (2004) "Central-Bank Interest-Rate Control in a Cashless, Arrow-Debreu Economy." Unpublished working paper, Department of Economics, Pennsylvania State University.

White, Lawrence H. (1984) "Competitive Payments Systems and the Unit of Account." *American Economic Review*, 74, 699-712.

Woodford, Michael. (2003) *Interest and Prices: Foundations of a theory of monetary policy*. Princeton: Princeton University Press.

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[1] The viability of e-cash systems that do not clear through the central bank is subject to debate (see e.g., Goodhart (2000), King 1999 and Friedman 2000). Friedman (2000) worried that as a consequence of the e-revolution banks would no longer need to clear through the central bank so central banks would lose their ability to control the policy rate. Goodhart (2000) offers an alternative perspective. However, such outcomes would not be equivalent to Woodford's cashless or frictionless model because they all rely on the existence of e-cash as a means of final settlement. At present such systems remain hypothetical possibilities because in e-cash channel systems banks are legally required to clear through the central bank.

[2] Recent examples that illustrate the futility of the Black-Fama approach to monetary theory are Wallace (2004) and Cochrane (2005).

[3] The active variation in the spread, the upper and lower bounds of the channel, to which Woodford refers here is not something that central banks have usually considered, but Goodhart (2009) explains how such variations can provide a useful additional instrument for liquidity management. See also Keister, Martin and McAndrews (2008) for an explanation of how the Federal Reserve has implemented such an arrangement.

[4] In this respect Woodford's model is quite different from Wicksell's pure credit model where ownership of the means of final settlement, credits, is tracked by book entry. Wicksell's model could be applied to understand the channel system, while Woodford's model cannot.

[5] See in particular Hahn (1982, p. 1) who explains why such systems have no role for money, or in this context, e-cash.

[6] Ljungqvist and Sargent (2004, p. 217, emphasis added) explain that these assumptions are equivalent to assuming what they call a time-0 auction.

"A symptom of the once-and-for-all trading arrangement is that each household faces one budget constraint that accounts for all trades across dates and histories." The time-0 auction removes all the frictions that money was invented to reduce including the friction of time.

[7] In this respect Woodford (2003, p.66) has two variables that he identifies as the nominal rate of interest. One labelled, i_t , appears in the definition of the discount factor Q_t and the other, i_t^m is the rate paid on money or cash balances. Clearly, when $M_t = 0$ the rate on money balances is not defined so the rate in the discount factor is no longer a nominal magnitude.

[8] Rogers (2006) explains how Hahn's analysis reveals why money or cash can always be treated as an inessential or redundant asset in Woodford's model.