Money as Friction: Conceptual dissonance in Woodford's *Interest and Prices*

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Abstract:

In *Interest and Prices* Woodford employs a frictionless model to derive nominal interest rate rules that can be applied by central banks to achieve price level stability. But frictionless models are Walrasian general equilibrium models that preclude any role for money. Furthermore frictionless model have no role for nominal values or the price level and therefore no role for a central bank. Consequently, conceptual anomalies arise in Woodford's attempt to analyse questions of monetary theory and policy that are precluded by construction in frictionless models. In some states of the model money is converted into a 'friction', contra economic theory.

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Introduction

A quick glance at the history of monetary thought reveals that money is understood to be an invention that reduces the frictions of barter and generally facilitates exchange and specialization. On this view, the fundamental property of money is that it expands trade and production activity- money is a welfare enhancing innovation; a lubricant not a friction.

Early general equilibrium theorists such as Patinkin (1965) and Clower (1967) accepted this view in their attempt to integrate money into general equilibrium models -the so-called integration of money and value theory. But it is now well understood that attempts to include money in the utility function or as a cash-in-advance (arrears) constraint fail to give money any essential role in Walrasian general equilibrium models. Hence it is not surprising that in later general equilibrium literature, money is seen as a complication that is best excluded from the model[1]. The idea is that the frictionless, "barter" models familiar from financial economics can be applied directly to questions of monetary theory and policy to generate testable predictions. Writers in this tradition are Black (1970), Fama (1980) and Lucas (1980, 1984). Contemporary exponents of this approach are Woodford (2003) and Cochrane (2001, 2005). But

[1] As Lucas (1984, p. 32) puts it when discussing the role of money in financial and monetary models: "..these observations do not amount to serious criticism of the application of a "barter" model to a monetary world. On the contrary, successful empirical applications of financial theory that abstracts from monetary complications testify to the good judgement of financial theorists in leaving such complications aside. The virtue of introducing monetary complications as done here is not to show that they affect the predictions of the theory (how could it be otherwise) but to show that they do so in a fully operational testable way".

Woodford and Cochrane go beyond the objective of *testable predictions* and propose *theoretical foundations* for monetary theory and policy. It is here that they run into difficulties. The most startling property of these models is that the introduction of a cash-in-advance (CIA) constraint converts money into a friction, *contra* economic theory.

The statement of his intentions by Woodford (2003, pp. 61-62, emphasis added) exposes the conceptual dissonance that results from attempts to apply frictionless models to monetary *theory*:[2]

"I first expound this approach in the context of a *purely cashless economy* –one in which there are *assumed to be no transactions frictions that can be reduced through the use of money balances*, and that accordingly provide a reason for holding such balances even when they earn a rate of return that is dominated by that available on other assets. Such a setting – one that is *commonly assumed in financial economics and in purely real models of economic fluctuations alike*.... At the same time, neither the usefulness nor the validity of the approach proposed here depends on the claim that *monetary frictions* do not exist in actual present-day economies. After expounding the theory for the cashless case, *I show how the framework can easily be generalized* to allow *for monetary frictions* that are common in monetarist models of inflation determination (by including real balances in the utility function or a cash-in-advance constraint). "

Here Woodford clearly identifies the traditional view of money as something that overcomes transactions frictions but he wishes to restrict his attention, initially, to a world were such frictions don't exist so there is no reason to hold money. But then he proposes to generalize his model to allow for 'monetary frictions' of the type said to be found in 'monetarist' models[3]. At face value this looks like a puzzle: how it is it possible, contra monetary economics 101, for money to become a friction?

[2] In similar vein, Cochrane (2005, p. 503) argues that the *cashless, frictionless fiscal model* can provide a useful benchmark for more complex and realistic analysis with frictions.
"Throughout economics, frictionless competitive models are the benchmark, the foundations upon which we add interesting frictions." See Rogers (2007) for a critique of Cochrane's model.
[3] However it is doubtful that monetarists would accept the Arrow-Debreu world as a suitable theoretical home.

This paper explains how this puzzle arises by outlining the conceptual anomalies that arise from the application of the frictionless, moneyless GE models with particular reference to Woodford (2003). It highlights the inability of these models to explain such concepts as nominal values, interest rates or prices; the concept of the price level, and electronic money (e-money). Inevitably these models fail to provide any theoretical foundations for monetary theory, interest rate rules or monetary policy in general.

The remainder of the paper proceeds as follows. Section 1 outlines examples of conceptual dissonance that arise when frictionless models are applied to monetary theory. Section 2 illustrates how the same examples appear in Woodford's (2003) model. Section 3 concludes.

1 Conceptual dissonance in frictionless models of money

Problems of conceptual dissonance arise for the interpretation of frictionless models because the meaning of familiar terms and concepts undergo subtle by crucial changes when employed in frictionless models. Important examples are the distinction and interpretation of *numeraire* and *nominal* prices, the meaning of the price level, the concept of money itself and the role of a central bank.

As many theorists are fond of claiming, the best-developed model of the economy is the Arrow-Debreu model[4]. The embarrassment for theorists is that the Arrow-Debreu

[4] For example, Wallace (2004) claims that the Arrow-Debreu model is the developed part of economics while monetary theory is undeveloped.

model has no role for money and the medium of exchange function is redundant. Hahn (1982, p. 1, emphasis added) put it succinctly:

"The most serious challenge that the existence of money poses to the theorist is this: the best developed model of the economy cannot find room for it. The best-developed model is, of course, the Arrow-Debreu version of a Walrasian general equilibrium. A world in which all conceivable contingent future contracts are possible neither needs nor wants intrinsically worthless money. …. The point is obvious and has been made quite often. *But it is doubtful that it has been fully taken on board*."[5]

Hahn's warning has gone unheeded and the conceptual dissonance surrounding the use of frictionless Walrasian general equilibrium models by monetary theorists is the consequence.

The most fundamental example is the conversion of money into a friction when the CIA constraint is imposed on agents in a frictionless world. To his credit Clower (1984, p. 267) realised that something was wrong when the CIA constraint was imposed on a frictionless Walrasian general equilibrium model:

"…the choice alternatives confronting households were more restrictive in a money than in a barter economy, which meant that monetary exchange is less efficient than barter exchange, contrary to both common sense and two hundred years of conventional wisdom. Something was wrong. But what?"

The simple answer to Clower's question is that the Walrasian or Arrow-Debreu contingent claims models are based on a recontract or time-0 auction that reduces them to models of *efficient* or *perfect barter*, as opposed to *real world barter* and its associated frictions and transactions costs that are reduced by the medium of exchange function of money. A frictionless model is frictionless because it is based on a time-0 auction of the type described by Ljungqvist and Sargent (2004, p. 208; p. 217 emphasis added):

[5] Sargent (1987, p. 133) makes the same point: "In equilibrium, an inconvertible currency is valueless; this result generally obtains in Arrow-Debreu models".

In the competitive equilibrium, all trades occur at date t = 0 in *one market*. Deliveries occur after t = 0, but no more trades. *A vast clearing or credit system* operates at t = 0. It ensures that [the budget constraint] holds for each household i. *A symptom of the once-and-for-all trading arrangement is that each household faces one budget constraint that accounts for all trades across dates and histories.*

The frictionless model is therefore a model of perfect barter rather than real world barter as noted by McCallum (1985, 2003). Imposing a medium of exchange function of money on such a world converts money from a welfare improving innovation in the real world into a welfare reducing friction *in the model*. The recontracting or time-0 auction performs (or eliminates the need for) the medium of exchange function of money so attempts by Patinkin and Clower to impose that function on a model where it is not required produced the inevitable counter intuitive result noted by Clower (1984).

Numeraire vs nominal prices and the meaning of the price level in frictionless models

A monetary economy is one where the three functions of money, medium of exchange, *numeraire* and store of value all attach to the same object. *Nominal* and *numeraire* values coincide in a monetary economy. McCallum (1985) explains why this is the case and why attempts to strip out or separate the functions of money invariably fail. By contrast the time-0 auction effectively removes any need for the medium of exchange function of money; the *numeraire* function can be assigned to anything, even something that does not exist; while the store of value attaches to the asset with the highest expected return.

In a monetary economy by contrast, Laidler (1990) has rightly pointed out that money exists because the Walrasian or time-0 auction does not. Nominal values exist because a medium of exchange exists that can be used to make payments and extinguish debts.

Money is the amazing invention that has allowed society to overcome the frictions of barter and enabled specialization in production and exchange. That is why money has utility! By embracing frictionless models monetary theorists have thrown this fundamental truth out the window. Consequently they find themselves caught in a web of conceptual anomalies.

The fact that nominal magnitudes and the price level are concepts that take on new meanings in the frictionless world under a time-0 auction has been recognised by a number of authors over the years. Hoover (1988) when discussing Fama's (1980) earlier attempt to apply the 'theory of finance' (frictionless models) to monetary theory, notes that Fama uses the term 'price level' to describe the *numeraire relative price* of commodities. For example, if oil is the *numeraire* then the *numeraire* price of all other commodities in terms of oil is a commodity relative exchange vector (so many grams of butter, jam etc per litre of oil with the litre of oil as *numeraire*). The nominal price, $'s per litre of oil is only of interest if we can part with the $s to acquire the oil. But if the medium of exchange function of money has been eliminated there are no nominal prices. Also, nominal magnitudes have meaning in a world where the purchasing power of money needs to be maintained so as to sustain the gains from trade and specialization that money makes possible. In other words, an economy with a medium of exchange needs to pay attention to its general purchasing power. But in a world where the time-0 auction can spit out the exchange value of an endowment in terms of any commodity as the *numeraire*, even one that has no physical existence, the notion of a 'price level' from a monetary economy is not defined.

Buiter (2002, 2004, p. 31) is another who has consistently argued that claiming to determine the price level in the frictionless model is akin to claiming the existence of phlogiston (the imaginary substance once thought to cause combustion):

"Any two commodities priced in phlogiston (or any imaginary and non-existent numeraire) will have a well-determined *relative* price. *Determining the price of phlogiston (the numeraire) when phlogiston does not exist except as a word, is an intellectual bridge too far*."

Furthermore, there is no theory of *numeraire* prices as Patinkin (1965) explained and Buiter (2007) now wonders if modern monetary theory based on frictionless models is degenerating to *numerairology*! Put simply, there are *no nominal values in frictionless models* and there is no theory of *numeraire* values.

The changing concept of money: fiat money vs asset money

The fact that the medium of exchange has no role to play in a frictionless model is also associated with a change in the concept of money in frictionless models. For money to have a non-zero value in the cashless, frictionless Walrasian world of Arrow-Debreu, it must be an income-earning asset. As Sargent (1987, p. 136) explains for the case of a Lucas tree model:

"In this economy, assets are valued according to the value of the stream of consumption that they support. An unbacked inconvertible currency promises to pay off nothing in the future. We have seen that introducing an asset with such a payoff stream into Lucas's tree model leaves the equilibrium interest rates unaltered and causes the asset to receive a zero value".

Hence in the application of finance theory to monetary economics the concept of money is changed. Unbacked fiat money is dropped and replaced by money as an income-earning asset –money as stock as Cochrane (2005) puts it. But these 'monetary assets' do not serve as the medium of exchange. Consequently frictionless

models cannot provide a theory for an economy with unbacked fiat money or one that requires any medium of exchange!

These examples of conceptual dissonance between frictionless models and the world of money and banking lead in turn to confusion about the relationship between the properties of frictionless models and the properties of modern e-money systems.

The time-0 auction is NOT equivalent to or a suitable proxy for e-money

Several economists and bankers, including Mervyn King, now Governor of the Bank of England, have speculated that the evolution of e-money means that the real world is converging on the properties of the pure barter exchange economy of economic theory. King (1999, p. 48) asked the question:

"Is it possible that advances in technology will mean the arbitrary assumptions necessary to introduce money into rigorous models will become redundant, and that the world may come to resemble a pure exchange economy? Electronic transactions in real time hold out that possibility."

Similarly, Green (2005, p. 31) interprets Woodford's frictionless model to be of direct relevance to central bankers:

"From the perspective of central bank economists, it is of great value to have a family of tractable models that yield intuitively appealing policy alternatives as optima."

The answer to King's question is no and this raises serious doubts about Green's interpretation of Woodford.

As several authors have noted, King's conjecture overlooks the fundamental difference between monetary economies and frictionless pure barter models (the pure exchange economy). The need for the distinction is simple: *a super computer*,

conducting e-money or e-asset trading in real time cannot approach the properties of a time-0 auction. Hoover (1988, p. 97) makes the relevant point that the adoption of e-fiat money and Real Time Gross Settlement (RTGS) systems does not mean that the real economy is converging on the properties of a world with a time-0 auction:

"The fact that computerization may allow us to dispense with notes and coins, does not transform our economy from one in which transactions are made in a higgledy-piggledy uncontrolled manner into one in which they are coordinated by central auction."

It is simply impossible to construct a computer that would be capable of collecting the information and coordinating anything remotely resembling a time-0 auction. For this reason King's (1999) concern is surely unfounded. For e-money of some form to approximate the time-0 auction would require the instantaneous flow and coordination of information that is beyond the capability of even the most super of computers –that is one reason why money exists – because the time-0 auction cannot. In short, *the time-0 auction is simply a non-operational thought experiment*. It is a *theoretical but non-operational substitute for money*. Conceptual problems emerge if the time-0 auction is treated as if it were a reasonable approximation to or proxy for the current or future e-payments system as some have suggested.

What is a central bank doing in a frictionless world?

The confusion between the evolution of e-money and the properties of the frictionless model also spills over into the role of the central bank in such models. Clearly there can be no role for a central bank in model based on a time-0 auction –there is simply nothing for it to do in such a world. Introducing another super agent in addition to the auctioneer running the time-0 auction will produce conceptual anomalies. In this respect it is useful to draw attention to the analysis by Wallace (2004) which highlights the futility of any attempt to find a role for a central bank in the cashless,

Arrow-Debreu economy. It is clearly going to be a challenge to prove the existence of a 'competitive equilibrium' in a model that attempts to introduce a monopolist price setter –for that is precisely what a central bank is, as Freedman (2000) and others have explained. Ultimately the model underlying Woodford's *Interest and Prices* is the cashless Arrow-Debreu world of Wallace (2004) who concedes that attempting to incorporate a central bank produces unsolvable puzzles.

To sum up, the cashless, frictionless economy is just too good to be true. It is an imaginary world in which all the issues of interest to monetary economists are eliminated by the time-0 auction. Nominal magnitudes do not exist, the concept of the price level is redundant and there is by implication then no scope for monetary or fiscal policy to determine it. Rogers (2007) outlines the futility of attempts to demonstrate the fiscal theory of the price level in a frictionless model as proposed by Cochrane (2005). The Arrow-Debreu model, which has these properties, is a theoretical construct that no doubt has its uses – but application to monetary theory is not one of them. Those who apply the cashless, frictionless model to analyse existing e-money systems generate an array of conceptual puzzles.

2 Conceptual dissonance in *Interest and Prices*

Over the past decade Woodford has offered two versions of the cashless, frictionless economy. The first, in Woodford (1998), employs the concept of a cash-in-arrears constraint that is made to disappear in the cashless limit. (Rogers 2006) presents a critique of this version[6]. The second version of a cashless economy is the cashless,

[6] Woodford (1998) distinguished between two forms of credit –informal and formal. Formal credit applies a *cash-in-arrears constraint* that disappears in the cashless limit. The intention is to model the evolution of money from cash to cashless (electronic money). However, informal credit is never defined but appears to be a synonym for the time-0 auction.

frictionless world presented in *Interest and Prices* where Woodford is quite explicit about the strategy he adopts.

Woodford (2003, p. 62, emphasis added) describes his vision of the frictionless model in the following terms:

"…considering *price-level determination* in an economy in which *both goods markets and financial markets are completely frictionless*….*Under the assumption of frictionless financial markets, it is natural to suppose that no 'monetary assets' are needed to facilitate transactions*".

Woodford (Woodford, p. 63) goes on to explain that:

"This is the approach that is taken in the chapters to follow. The basic model (developed beginning in Chapter 2) is one that abstracts from monetary frictions…."

"As we shall see, the central bank's policy rule is one of the key determinants of the equilibrium price level even in a cashless economy".

The clear intention is for the central bank to determine the price level in the cashless or frictionless version of the model. It view of what was said in section 1 this will be a challenge. It is also Woodford's intention to treat the cashless or frictionless model as a limiting state of an economy with 'monetary frictions'.

After expounding the theory for the cashless case, *I show how the framework can easily be generalized to allow for monetary frictions…"*

Recall that Woodford has here inadvertently converted money from something that overcomes frictions in the real world into something that is the source of friction in his model. Woodford is forced into this subtle change in the role of money because he employs the frictionless model as the base-line model into which 'monetary frictions' will, at times, be incorporated. Woodford also assumes:

"…*complete financial markets,* that is, the available financial assets completely span the relevant uncertainty faced by households about future income, prices, taste shocks and so on *so that each household faces a single budget constraint*".

This is the version of the frictionless world familiar from financial economics that is isomorphic with the Arrow-Debreu model.

As Woodford is advocating a cashless or frictionless world as a benchmark that is easily generalized to incorporate 'monetary frictions' he retains money (cash) and nominal interest rates in the household's one-period budget constraint in the 'monetary frictions' version of the model (equation numbers follow Woodford 2003):

$$P_t C_t + \Delta_t M_t + E[Q_{t,t+1} W_{t+1}] \leq W_t + [P_t Y_t - T_t] \qquad (1.7)$$

where $\Delta_t \equiv \frac{i_t - i_t^m}{1 + i_t}$. The variables are defined as follows: The variable M_t is the end of period holding of money base, while $E_t[Q_{t,t+1} W_{t+1}]$ is the stochastic present value of monetary and non-monetary wealth where $W_{t+1} = (1 + i_t^m) M_t + A_{t+1}$ and A_{t+1} is the random value of non-monetary wealth. P_t is the price of the consumption good C_t in terms of money and Y_t is the possibly stochastic endowment of the single good. T_t is net nominal tax collected by the government. Woodford then argues that the infinite sequence of budget constraints in (1.7) *is equivalent to a single inter-temporal budget constraint (1.12).*

$$\sum_{t=0}^{\infty} E_0 Q_{0,t} [P_t C_t + \Delta_t M_t] \leq W_0 + \sum_{t=0}^{\infty} E_0 Q_{o,t} [P_t Y_t - T_t] \qquad (1.12)$$

Woodford goes on to argue that, in the frictionless economy, there is no non-pecuniary benefit from holding money balances so household optimization requires that:

Either, $M_t = 0$ **or** $i_t = i_t^m$;

at each date and in each state (but which condition obtains may differ across dates and states). So in the 'monetary frictions' state of the model $M_t > 0$ while in the frictionless state of the model, money or cash has been eliminated so $M_t = 0$. As explained in section 1 this must raise questions about the interpretation of Woodford's model because the medium of exchange function of money has been eliminated.

Interpreting Woodford's model

The first thing that strikes the reader is that in its frictionless state Woodford's model is based on a time-0 auction. Recall that the ability to construct a single budget constraint like (1.12) is a property of the time-0 auction as noted by Ljungqvist and Sargent (2004, p. 217, emphasis added).

A symptom of the once-and-for-all trading arrangement is that each household faces one budget constraint that accounts for all trades across dates and histories.

As it is implicitly based on a time-0 auction Woodford's frictionless model therefore has no role for nominal values or the price level and certainly no role for the central bank to manipulate a nominal interest so as to stabilise the price level. Woodford's attempt to apply the frictionless model to analyse issues which it precludes by construction inevitably leads to conceptual puzzles. The discussion that follows illustrates the following properties of Woodford's frictionless model:

(i) There are no nominal values.
(ii) There is no medium of exchange and no role for the price level.

(iii) Fiat money disappears and is replaced by an 'income-earning' monetary asset.
(iv) There is no way for a central bank to determine any interest rate.

(i) No nominal values exist in the frictionless state of the model

Consider the variables i_t^m and i_t as defined by Woodford. The former is defined as the nominal rate of interest paid on money balances held at the end of the period, *t*. Presumably, if money balances are zero we are in the cashless, frictionless state and i_t^m is not defined, leaving only what Woodford also calls the 'nominal' interest rate, i_t.[7] However, if there is no money (cash) how can the 'nominal' interest rate i_t exist when the nominal rate oh cash, i_t^m does not? Obviously it cannot so we have two variables parading as nominal interest rates in different states of Woodford's model. One disappears when cash is eliminated in the frictionless state but the other remains. What exactly is i_t when cash has been eliminated?

In the frictionless state of Woodford's model the 'nominal' rate of interest, i_t, is defined as the short term 'nominal' interest rate that could solve the equation $A_{t+1} = (1+i_t)B_t$ where B_t is the nominal value (in terms of *numeraire*) of household end-of-period portfolio of other financial wealth (excluding money). But this begs the question- what does 'nominal' mean in this context? A nominal interest rate is nominal because it implies a calculation using an interest rate that produces a *quantity of cash* (money) for future use. When *the $ is the medium of exchange and the numeraire* we describe the interest rate on lending cash as the nominal interest rate.

[7] In the case where money balances are zero the term $\Delta_t \neq 0$, ie $\Delta_t = \frac{i_t}{1+i_t}$.

When cash has been eliminated but the *$* remains only as a *numeraire* and has no physical existence the fact that the amount of interest earned on a non-monetary asset is calculated in *$s* does not mean that it is a nominal interest rate. In the cashless, frictionless world the choice of *numeraire* is entirely arbitrary-it could be jam or phlogiston. We would not describe the jam or phlogiston rate of interest on a bond as a nominal interest rate but this is implicitly what Woodford asks us to do. Woodford is confusing *numeraire* prices with money or nominal prices.

Woodford then compounds the confusion by defining the *numeraire* in terms of a *quantity* of the liability of the central bank even when the *numeraire* may not exist, Woodford (2003, p. 63, emphasis added): [8]

> "However, I assume *that there exists a monetary unit of account in terms of which prices (of both goods and financial assets) are quoted***.** This unit of account is defined in terms of a claim to *a certain quantity* of a liability of the central bank, *which may or may not have a physical existence".*

If the unit of account does not have a physical existence, then it cannot be defined in terms of the *quantity* of the liability of the central bank, as Woodford claims. If phlogiston is the unit of account, as Woodford allows, in what units is it measured? This is obviously a slip. But it is a slip that is inevitable when attempting to apply a frictionless Walrasian general equilibrium model to monetary theory.

(ii) There is no medium of exchange and therefore no role for the price level

The confusion between nominal and *numeraire* prices arises because the medium of exchange function of money has been eliminated. From Woodford's description of

[8] It should be noted that there is no reason why, in the model, the 'thing' that serves as the unit of account should be restricted to Woodford's monetary asset.

the cashless, frictionless model it is apparent that even e-asset trading has no role –no 'monetary assets' are needed to facilitate transactions. Recall Woodford's (2003, p. 62) claim that:

Under the assumption of frictionless financial markets, it is natural to suppose that no 'monetary assets' are needed to facilitate transactions".

It is only 'natural' to make this supposition in a model with a time-0 auction. No central banker would ever contemplate it.

The conceptual difficulties faced by Woodford's abolition of any medium of exchange are also exposed by Green's (2005, p. 124) description of Woodford's model:

"In the cashless economy that Woodford models, *the central bank can set the a nominal interest rate directly by standing ready either to lend or borrow unboundedly at its policy interest rate* (analogous to the Federal funds rate in the United States). However, the outside *money* that is borrowed or lent *is a redundant asset in a complete system of markets* for dated, state-contingent consumption, *and plays no essential role in facilitating transactions."*

Green correctly identifies money as a redundant asset that plays no role in facilitating transactions in Woodford's model–there is no medium of exchange in the cashless or frictionless version of the model. The description of the monetary asset as 'redundant' raises the question about its relevance to the model. [9]

If the goods 'markets' are frictionless then they have the properties exhibited by the time-0 auction and the model has nothing to say about how those exchanges will be

[9] A related conceptual puzzle is raised by Woodford's claim that the 'monetary asset' is a perfect substitute for *other riskless nominal assets of similarly short maturity*. But in a fiat money economy with a channel system the central bank is a price setter not a price taker. If the 'central bank' is dealing in a monetary asset that has perfect substitutes monetary policy as implemented in RTGS systems is obviously impossible.

executed. Commodities and assets trade directly in that world so there is no need for e-asset money let alone e-fiat money. Under a time-0 auction there is also no need to distinguish the asset or commodity composition of the household endowment. All commodities and assets are on an equal footing and all can be traded directly under the time-0 auction –that is why it is a cashless or frictionless world. As Lucas (1984, p. 10, emphasis added) notes:

"*A central feature of this model is that all trading occurs in a central market, with all agents present. In such a setting, the position of each agent is fully described by a single number: his wealth, or the market value of all the claims he owns (endowment). The command of any one claim over goods is fully described by its market value, which is to say all claims are equally liquid*."

Such a frictionless world has no role for money of any sort, be it electronic or cash; money can serve no useful purpose. Money has no utility in such a world and this explains the failure of Patinkin's (1965) attempt to incorporate money in the utility function in what amounts to a frictionless Walrasian general equilibrium model. The source of the difficulty has been well documented in the literature since Hahn (1965). The medium of exchange has no role and the concept of a general price level is redundant – agents have no use for it in a frictionless world. All they require is the bi-lateral commodity exchange ratios.

(iii) The changing concept of money in Woodford's model

As with the dual nature of the 'nominal' interest rate the concept of money also changes between the two states of Woodford's model. In the cashless or frictionless state of the model money is treated as an income-earning asset as explained by Sargent. Unbacked fait money has zero exchange value under a time-0 auction unless additional constraints are placed on agents. The CIA constraint is an example of such

a constraint so when Woodford deals with 'monetary frictions' he in fact imposes those frictions on agents so as to enforce the use of unbacked fiat money.

Thus when the model is in the frictionless state $M_t = 0$ and an income-earning 'monetary asset' is treated as money but when $M_t > 0$ the use of unbacked fiat is imposed on agents by the CIA constraint and money becomes a friction. There is no real economy where money under goes such a transformation so the claim that the frictionless model is *easily generalised* to incorporate 'monetary frictions' must be rejected as a relevant modelling strategy. Furthermore, it is not the case that the medium of exchange is disappearing as Woodford often suggests. The medium of exchange is evolving from cash to electronic transfer. It is also not the case that 'income-earning monetary assets' will become the universal medium of exchange in the electronic age. Fiat e-money will continue to dominate such assets by reducing the frictions that the use of 'monetary assets' would imply. Rogers (2007) explains why Cochrane's claim that 'stocks' can become the universal medium of exchange should be rejected.

(iv) There is no role for the central bank

Finally, it follows from what has been said previously that if there is no role for nominal rates of interest, the price-level and the medium of exchange there is also no role for the central bank.

In chapter 1 of *Interest and Prices* Woodford describes what he calls the 'channel system' of interest rate control. The 'channel system' refers to Real Time Gross Settlement (RTGS) or e-money systems as implemented by the central banks of Australia (Campbell 1988), New Zealand and Canada, among others. It is clearly

Woodford's (2003, pp. 63-64, emphasis added) intention to provide a theoretical model for such a 'channel system' when he argues:

"*…the central bank's policy rule is one of the key determinants of the equilibrium price level even in a cashless economy,…* But in such a world, *the crucial tool* available to the central bank *will not be open market operations, but the possibility of adjusting the interest rate paid on central bank balances*".

Woodford (2003, p. 32, emphasis added) insists that in such a system a clearing demand at the central bank is not required:

"But once the idea has been accepted that the central bank can vary the overnight interest rate without ever having to vary the size of the return spread, *the functioning of the system no longer depends on the existence of a clearing demand***."**

This statement is simply incorrect as an explanation of how modern 'channel' or RTGS systems work. RTGS systems have the following characteristics that are relevant to the assessment of Woodford's analysis.

As explained by Freedman (2000) and others, the key feature of the 'channel system' is that the central bank is the monopoly supplier of clearing balances and all financial institutions and banks clear directly or indirectly through the central bank. The central bank sets what Woodford (2003) calls 'the channel', the spread between the interest rate paid on surplus clearing balances and the penalty rate paid if there is a shortfall of clearing balances. The 'channel' or spread is set to discourage the banks from holding surplus clearing balances at the central bank and to induce the banks to manage their daily and hourly cash flow to avoid the penalty rates on any clearing balance shortfall. As Campbell (1998) explains, on a daily and hourly basis (real time), the central bank engages in what we can call open market operations (mostly repurchase agreements) to ensure that the supply for clearing balances matches the demand at the target interest rate. Ensuring there are no surplus or deficits of clearing balances in real time

gives the central bank tight control over the target rate. Examination of the short-term deviations between market overnight rates and the bank rate indicates that central banks and commercial banks have become very adept at managing this system with the virtual elimination of deviations between the overnight rate and the target rate. See Figure 1.1 in Woodford (2003, p. 1.1), which illustrates the Canadian system.

For this system to work there must exist a medium of exchange or final settlement of which the central bank is the monopoly supplier. Thus a central bank that is operating effectively is always in a position to impose losses on individual banks that do not manage their cash flows effectively or on the banking and financial system as a whole when it seeks to raise the target rate. All banks are aware of this and that is why effective central banks' announcements of a rise in the target rate elicit an immediate response without the need for reinforcement by massive open market operations. Woodford (2003, pp. 35-37) accurately describes many of these properties, subject to the caveat that these systems *do* rely on the existence of a clearing demand at the central bank.

Consequently for Woodford's 'channel' system to operate effectively, the medium of exchange function for money must exist because without this function the central bank has no mechanism to achieve and enforce the target nominal rate of interest. Woodford (2003, p. 31) appears not to fully appreciate the significance of this point. He provides two reasons for using the frictionless model as a model of the 'channel' system. Neither of these reasons withstands careful evaluation.

The first reason he gives refers to the possibility that electronic payments systems may eliminate any advantage of clearing through the central bank - an idea due to

King (1999) – and the suggestion by Benjamin Friedman (2000) that under such circumstances monetary policy would fail because the central bank would be like an "army with only a signal corps". There are two ideas conflated here.

The first is that the modern electronic money system will evolve to emulate the properties of the real Walrasian, Arrow-Debreu barter or pure barter exchange economy under a time-0 auction. The discussion in section 1 explains why that cannot happen. Second, is the idea that with the emergence of e-money, banks will no longer clear through the central bank but form their own clearing-house. If such a system were to occur it would certainly compromise the ability of the central bank to implement monetary policy, as we now know it. For that reason such development is prevented by legal restriction and in RTGS systems all banks and financial institutions must clear through the central bank. Despite this legal restriction it could be argued that the single clearing-house model – a central bank- is more efficient than alternative systems and will survive the electronic revolution as argued by Freedman (2000) and others. In any event those who argue that e-money could evolve in the way imagined by King or Friedman are effectively arguing that some form of free banking is practically possible in a purely private e-money system. By contrast, exponents of free banking usually require conversion to some redemption medium such as gold or bonds to tie down the price level in traditional quantity theory fashion.

The second reason Woodford (2003, p. 32) gives is that it is possible to demonstrate effective interest rate control in the complete absence of monetary frictions, i.e., in the cashless or frictionless model. This claim has been dealt with in some detail above. In essence it cannot be substantiated on theoretical or operational grounds.

On theoretical grounds it is not possible to use frictionless models to show that nominal interest rate rules can be applied to determine the price level or stabilize inflation. Frictionless models are what they have always been; models of efficient or perfect barter that have no role for a medium of exchange of any form –cash or e-transfer. At best the time-0 auction can be interpreted as a substitute for money as suggested by Laidler (1990). On operational grounds Woodford's claim that modern 'channel' or RTGS systems do not operate on a clearing demand at the central bank is simply false. The existence of such a clearing demand at the central bank or clearing house is essential for central bank control of the interest rate. In some systems, control over the interest rate is facilitated by the movement of government balances between commercial (private) banks and the central bank. The key point is that it is the existence of a residual clearing demand at the central bank that enables it to impose, or threaten to impose, losses on the banks and that gives the central bank control of the target rate. Without such a mechanism the central bank is indeed an 'army with just a signal corps' and, paradoxically, that is effectively what Woodford's formal frictionless model is.

Thus, although Woodford (2003, pp. 24-37) accurately describes many of the properties of RTGS or 'channel systems' the properties of that system are not captured by his frictionless model where the medium of exchange function has been eliminated as this effectively eliminates any clearing demand at the central bank. Defining the unit of account in terms of the liabilities of the central bank, treating the 'monetary asset' as the unit of account, is not sufficient to give the central bank control of the target rate of interest and does not mean that the medium of exchange or clearing demand can be dropped.

Finally, it is clear that Woodford (2003. p. 35, emphasis added) is well aware that a medium of exchange and hence residual clearing demand is in fact necessary to the functioning of the Federal Reserve system:

"The answer is that the unit of account in a purely fiat system is defined in terms of the liabilities of the central bank. A financial contract to deliver a certain number of U.S dollars at a specified future date is promising payment in terms of Federal Reserve notes *or clearing balances at the Fed*".

Here Woodford acknowledges in the Federal Reserve system a medium of exchange exists, US dollars or the clearing balances at the Fed, and that this clearing demand is important in enabling the central bank to control the target rate, *contra* his earlier argument that clearing demand can be ignored *in the frictionless model*. In the frictionless model the monetary asset (base money) does not act as the medium of exchange so the model is incapable of capturing this essential aspect of monetary policy. In a modern cashless world of e-money, electronic transfers and clearing balances are the medium of exchange and modern RTGS or 'channel' systems rely on the existence of demand for clearing balances at the central bank for the control of the target rate. The frictionless model contains none of these features and cannot therefore provide the basis for the analysis of central bank nominal interest rate rules. The point here is that Woodford's use of the frictionless model inevitably creates conceptual dissonance between the properties of the frictionless model and e-money economies.

In conclusion, the frictionless, cashless world of Woodford is the world of Arrow-Debeu securities underpinned by the time-0 auction. It is the failure to recognise the implications of the time-0 auction that has produced the conceptual dissonance in Woodford's *Interest and Prices*. In the frictionless world we can apply the arbitrage

relations familiar from financial economics to find the implicit price of money that has been made to vanish is if money were an Arrow-Debreu security. Although this is a false description of a monetary economy Woodford compounds the confusion by retaining a role for cash (fiat or asset money) in some states of the model. In these states the model exhibits the Patinkin-Clower puzzle – money is a welfare reducing friction –contrary to 200 years of conventional wisdom and common sense. But more damaging is the fact that the frictionless world has no role for the medium of exchange function of money, nominal magnitudes - interest rates, prices and price levels – and therefore no role for central banks.

3 **Concluding remarks**.

In *Interest and Prices* Woodford attempts to square the circle by proposing to derive nominal interest rate rules that can be employed by a central bank to stabilise the price level or inflation in a frictionless model. He then proposes to generalise the model to allow for 'monetary frictions' of the kind found in the literature on CIA constraints. The first of these objectives is unattainable while the second is inconsistent with basic economic theory – money is a lubricant not a friction.

Frictionless models and Arrow-Debreu versions of Walrasian general equilibrium models rest on the time-0 auction. The time-0 auction is a device that at best can be interpreted as a means of avoiding all the complications associated with money as Lucas (1984) observed. Woodford searches for interest rate rules in a frictionless model without a medium of exchange and consequently empties the model of all nominal values in a world that has no role for the central bank. Frictionless models of the type employed by Woodford have nothing to say about monetary theory. By

ignoring this property of his frictionless model Woodford's analysis is characterised by the conceptual puzzles illustrated in this paper. In particular, 'generalizing' the frictionless model to allow for 'monetary frictions' then replicates the Patinkin-Clower puzzle of converting money into a 'friction'.

There is no escaping the conclusion that the time-0 auction is essential for the frictionless property of the model. Monetary theory cannot progress in the context of frictionless models under a time-0 auction. Instead, monetary theory requires the development of what Hahn (1973a,b) called an *essential monetary or sequence economy.* The latter is a world which exhibits all the transactions frictions removed by the time-0 auction. In theoretical terms it is a world of incomplete markets and search into which money enters as a welfare improving innovation that overcomes frictions – as conventional wisdom suggests. So the time-0 auction and the auctioneer must go to enter that world –as many have realised. It is rather like the world of monetary economics 101 and the world of e-money inhabited by central bankers. As outlined by Rogers (2007), the lesson to be learnt from attempts at monetary theory in frictionless models is simple: monetary theory does not map anywhere into frictionless models.

The challenge for monetary theorists is to produce an *essential sequence economy.* A model of incomplete markets and search to which the medium of exchange function of money can be incorporated as a welfare increasing innovation. Such a model should be the baseline model employed by Woodford.

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