Applied Economics Letters, 2007, **14**, 211–218

Crisis-induced intermittency in non-linear economic cycles

A. C.-L. Chian[a,b,*], E. L. Rempel[c] and C. Rogers[b]

[a]*National Institute for Space Research (INPE) and World Institute for Space Environment Research (WISER), P. O. Box 515, São José dos Campos-SP 12227-010, Brazil*
[b]*School of Economics, University of Adelaide, SA 5005, Australia*
[c]*Institute of Aeronautical Technology (ITA), São José dos Campos-SP 12228-900, Brazil*

A new type of economic intermittency is found in non-linear business cycles. Following a merging crisis, a complex economic system has the ability to retain memory of its weakly chaotic dynamics prior to crisis. The resulting time series exhibits episodic regime switching between periods of weakly and strongly chaotic fluctuations of economic variables. The characteristic intermittency time, useful for forecasting the average duration of contractionary phases and the turning point to the expansionary phase of business cycles, is computed from the simulated time series.

I. Introduction

Intermittency is a fundamental dynamical feature of complex economic systems. An intermittent economic time series is characterized by recurrence of regime switching between periods of bursts of high-level fluctuations of economic activities and periods of low-level fluctuations. For example, an instability of the financial system leads to speculative booms followed by subsequent financial crises manifested by violent price movements in financial markets; the recurrence of these events results in business cycles with alternating periods of boom and depression (Mullineux, 1990). The spectral density of intermittent economic time series indicates power-law behaviour typical of mutiscale systems. Statistical analysis of the high-frequency dynamics of stock markets and foreign exchange markets has proven the intermittent nature of these financial systems, which display non-Gaussian form with fat-tail in the probability distribution function of price changes (Mantegna and Stanley, 2000).

A good understanding of regime switching and memory of economic time series is essential for pattern recognition and forecasting of business cycles. Kirikos (2000) compared a random walk with Markov switching-regime processes in forecasting foreign exchange rates; the results suggested that the availability of more past information may be useful in forecasting future exchange rates. Kholodilin (2003) introduced structural shifts in the US composite economic indicator via deterministic dummies and evaluated the US monthly macroeconomic series specified by the regime-switching model. Bautista (2003) used regime-switching-ARCH regression on the Philippine stock market data to estimate its conditional variance and relate to episodes of high volatility including the 1997 Asian financial crisis; this study identified a period of high stock return volatility preceding a bust cycle marked by a sequence of low-growth periods. Granger and Ding (1996) defined long memory as a time series having a slowly declining correlogram, which is a property of fractional integrated processes as well as a number of

*Corresponding author. E-mail: achian@dge.inpe.br

Applied Economics Letters ISSN 1350–4851 print/ISSN 1466–4291 online © 2007 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/13504850500425436

other processes including non-linear models; the relevance of long memory is illustrated using absolute returns from a daily stock market index. Resende and Teixeira (2002) assessed long-memory patterns in the Brazilian stock market index (Ibovespa) for periods before and after the Real Stabilization Plan, and obtained evidence of short memory for both periods. Gil-Alana (2004) presented evidence of memory in the dynamics of the real exchange rates in Europe using the fractional integration techniques. Muckley (2004) employed rescaled-range analysis, correlation dimension test and BDS test to obtain evidence of long-memory effect and chaos in daily time series of financial data.

Intermittency is ubiquitous in chaotic economic systems. In a non-linear macroeconomic model (Mosekilde *et al.*, 1992) describing an economic long wave (or Kondratiev cycle) forced by an exogenous short-term construction (or Kuznets) business cycle represented by a sinusoidal fluctuation in the demand for capital to the goods sector, a chaotic transition known as crisis involving a sudden expansion of chaotic attractor and a complex form of chaos arising from intermittency are observed. In a disaggregated economic long-wave model describing two coupled industries (Haxholdt *et al.*, 1995), one representing production of plant and long-lived infrastructure and the other representing short-lived equipment and machinery, mode-locking, quasi-periodic behaviour, chaos and intermittency are detected. In a model of an economic duopoly game (Bischi *et al.*, 1998), the phenomenon of synchronization of a two-dimensional discrete dynamical system is studied and on-off intermittency due to a transverse instability is detected.

An example of type-I intermittency in non-linear business cycles was studied recently (Chian *et al.*, 2006). In the economic type-I intermittency, the recurrence of regime switching between bursty and laminar phases indicates that a non-linear economic system is capable of keeping the memory of its ordered dynamics after the system evolves from order to chaos due to a local saddle-node bifurcation. Most econometric studies of long memory treat economic data as stochastic processes (Granger and Ding, 1996; Resende and Teixeira, 2002; Gil-Alana, 2004), however, real economic systems are a mixture of stochastic and deterministic processes. In this article, we adopt the deterministic approach to study a new type of economic intermittency induced by an attractor merging crisis due to a global bifurcation (Chian *et al.*, 2005). We will show that following the onset of an attractor merging crisis, the economic system retains its memory of the weakly chaotic dynamics before the crisis; as the result, the time series of business cycles becomes intermittent displaying episodic regime switching between periods of weakly and strongly chaotic fluctuations.

A forced model of non-linear business cycles is formulated in Section II. Economic crisis-induced intermittency is analyzed in Section III. The conclusion is given in Section IV.

II. Non-linear Model of Business Cycles

We model the non-linear dynamics of business cycles driven by a periodic exogenous force using the van der Pol differential equation (Chian *et al.*, 2005, 2006)

$$\ddot{x} + \mu(x^2 - 1)\dot{x} + x = a\sin(\omega t), \qquad (1)$$

where x denotes an economic variable such as production, the dot denotes derivative with respect to time t, μ is an endogenous damping parameter, a denotes the driver amplitude and ω denotes the driver frequency. The driven van der Pol Equation 1 admits periodic (ordered) or aperiodic (chaotic) solutions as we vary any of the three control parameters: a, ω, μ. Equation 1 (when $a = 0$) is invariant under the flip operation ($x \rightarrow -x$). This symmetry is a typical property of dynamical systems that exhibit attractor merging crises (Chian *et al.*, 2005, 2006).

III. Economic Crisis-induced Intermittency

The qualitative structure of the trajectory described by Equation 1 can change (i.e. bifurcate) as the control parameters are varied. For example, fixed points can be created or destroyed, or their stability can change. These changes in the system dynamics can be represented by the bifurcation diagram. A periodic window of the bifurcation diagram determined from the numerical solutions of Equation 1 is shown in Fig. 1, where we plot $\dot{x}$ as a function of the driver amplitude for a while keeping other control parameters fixed ($\mu = 1$ and $\omega = 0.45$). Within the periodic window, two (or more) attractors A_1 and A_2 coexist, each with its own basin of attraction (Chian *et al.*, 2005, 2006). At $a = 0.98312$, a period-1 limit cycle for each attractor A_1/A_2 is generated via a local saddle-node bifurcation (SNB), which evolves into a small chaotic attractor via a cascade of period-doubling bifurcations.

An attractor merging crisis (MC) occurs at the crisis point, near $a = a_{MC} = 0.98765$. The phase-space trajectories of two small chaotic attractors

(CA_1 and CA_2) in the phase space $(x, \dot{x})$, near the crisis point, are shown in Figs 2(a) and (b), respectively. Note that CA_1 and CA_2 are symmetric with respect to each other. In fact, the dynamic properties of these two co-existing attractors are identical. At the crisis point, each of the two small chaotic attractors simultaneously collide head-on with a period-3 mediating unstable periodic orbit on the boundary which separates their basins of attraction, leading to an attractor merging crisis due to a global bifurcation (Chian *et al.*, 2005). As the consequence, the two pre-crisis small chaotic attractors merge to form a post-crisis large merged chaotic attractor (MCA), as seen in Fig. 2(c).

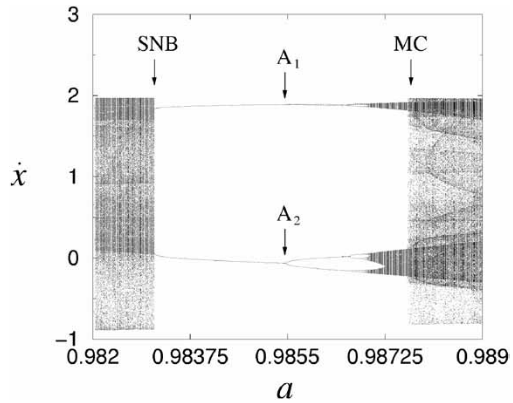

Fig. 1. Bifurcation diagram of $\dot{x}$ as a function of the driver amplitude *a* for attractors A_1 and A_2. MC denotes attractor merging crisis and SNB denotes saddle-node bifurcation. $\mu = 1$ and $\omega = 0.45$

A Poincaré map of the phase-space trajectories of Fig. 2 is plotted in Fig. 3, which is a superposition of two pre-crisis weak chaotic attractors (CA_1 and CA_2) and the post-crisis strong MCA. We define a stroboscopic Poincaré map

$$P: [x(t), \dot{x}(t)] \rightarrow [x(t+T), \dot{x}(t+T)], \qquad (2)$$

where $T = 2\pi/\omega$ is the driver period. Note that the two pre-crisis CA_1 and CA_2 are located in two small regions within the post-crisis MCA.

The time series of $\dot{x}$ for the two small chaotic attractors CA_1 and CA_2 at crisis, $a = 0.98765$, are shown in Figs 4(a) and (b), respectively. The same time series of Figs 4(a) and (b) plotted as a function of driver cycles are shown in Fig. 4(c). From Fig. 4(c),

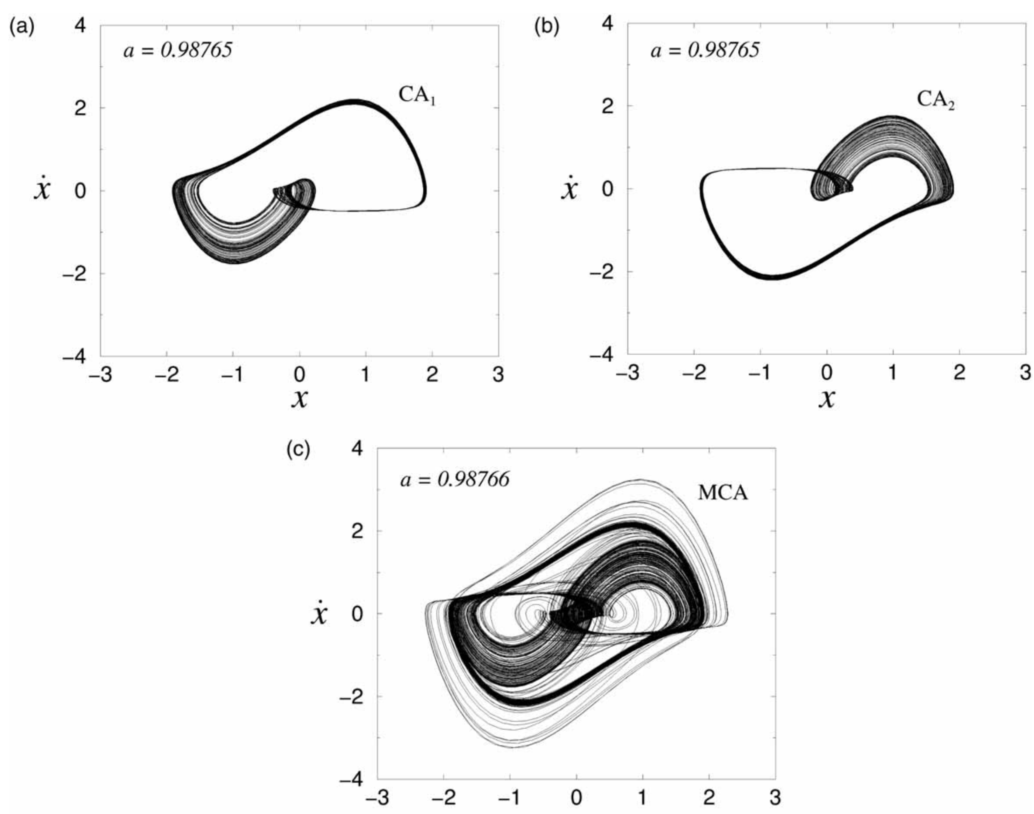

Fig. 2. Phase-space trajectories of: (a) pre-crisis chaotic attractor (CA_1) for $a = 0.98765$, (b) pre-crisis chaotic attractor (CA_2) for $a = 0.98765$, (c) post-crisis merged chaotic attractor (MCA) for $a = 0.98766$

we see that before crisis the fluctuations of economic variables are weakly chaotic (laminar), localized in a small range of $\dot{x}$ (near $\dot{x} \sim 2$ and $\dot{x} \sim 0$), consistent with the Poincaré map in Fig. 3.

After the attractor merging crisis, there is only one large chaotic attractor (MCA) in the system. The time series of $\dot{x}$ of MCA after the crisis, for $a = 0.98766$ and $a = 0.988$, are shown in Figs 5(a) and (b), respectively. The same time series plotted as a function of driver cycles are shown in Figs 5(c) and (d), respectively. The time series in Fig. 5 show that the system dynamics becomes intermittent after the onset of attractor merging crisis, with periods of weakly chaotic (laminar) fluctuations interrupted abruptly by periods of strongly chaotic (bursty) fluctuations. A comparison of the time series of Figs 4 and 5 indicates that the laminar phases in Fig. 5 are related to the pre-crisis attractors CA_1 and CA_2. Hence, the post-crisis system keeps memory of its weakly chaotic dynamics prior to crisis, and switches back and forth between the low-level fluctuations related to CA_1 and CA_2, linked by high-level fluctuations related to MCA. An examination of Fig. 5 shows that, as the system moves away from the crisis point, the average duration of laminar phases decreases and the regime switching becomes more frequent.

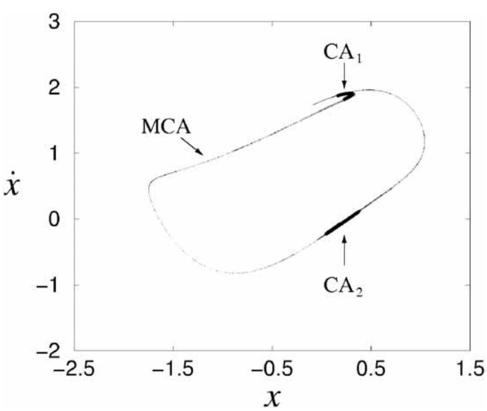

Fig. 3. Poincaré map of the post-crisis merged chaotic attractor (MCA, light line) for $a = 0.98766$, superposed by the pre-crisis chaotic attractors (CA_1 and CA_2, dark lines) for $a = 0.98765$

The power spectra associated with the time series of Figs 4 and 5 are shown in Fig. 6. It is evident that in all three cases the high-frequency portions of the spectra present power-law behaviours, which are

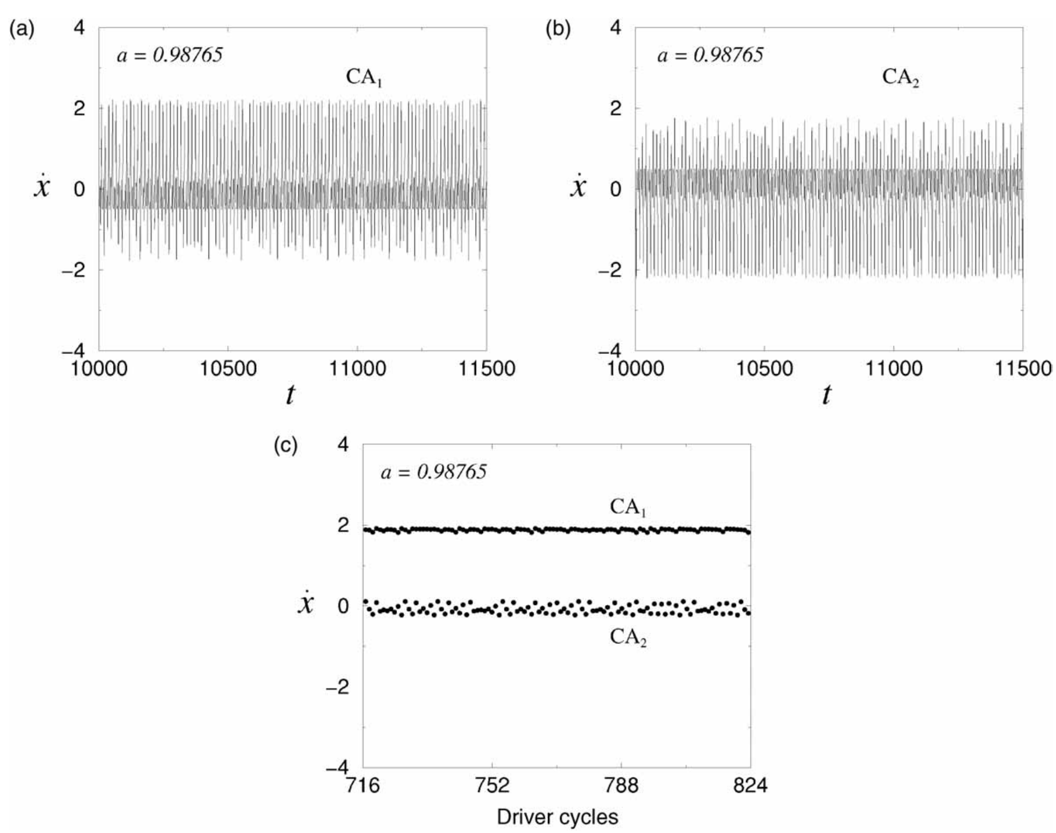

Fig. 4. Pre-crisis time series of $\dot{x}$ for $a = 0.98765$: (a) $\dot{x}(t)$ for chaotic attractor CA_1; (b) $\dot{x}(t)$ for chaotic attractor CA_2; (c) $\dot{x}$ as a function of driver cycles for (a) and (b), respectively

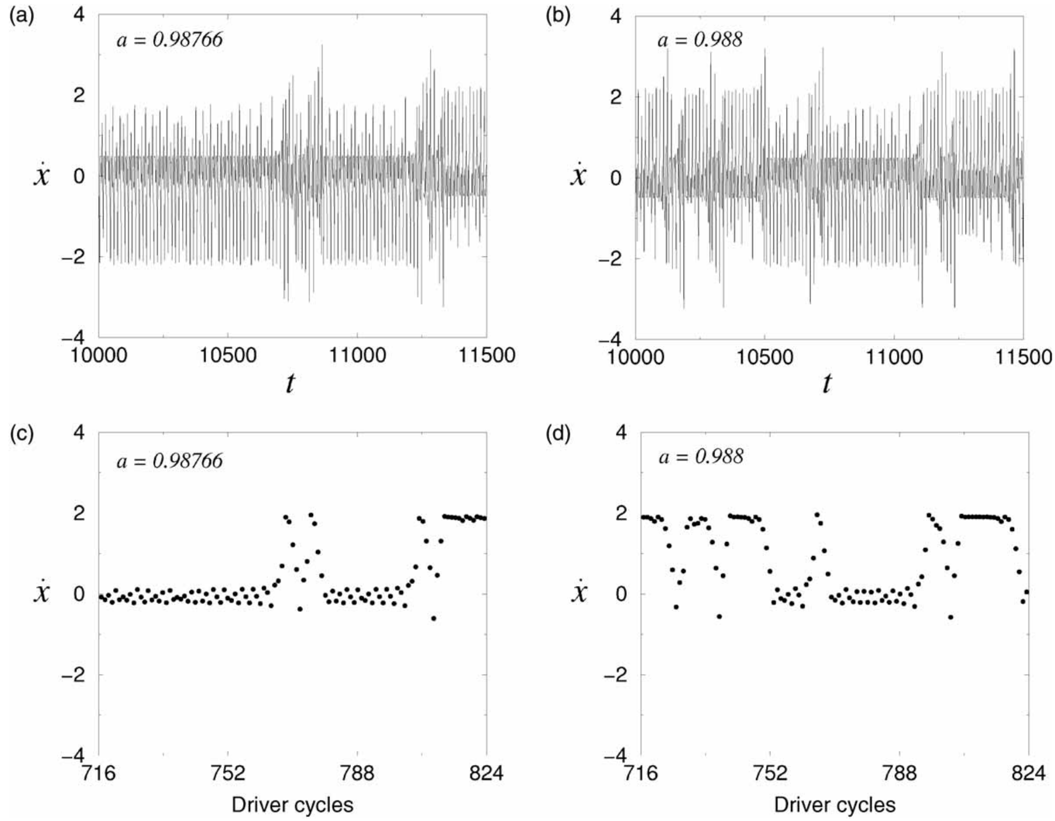

Fig. 5. Post-crisis intermittent time series of $\dot{x}$ for $a = 0.98766$ and $a = 0.988$. (a) and (b): $\dot{x}(t)$; (c) and (d): $\dot{x}$ as a function of driver cycles for (a) and (b), respectively

typical features of intermittent financial systems such as stock markets and foreign exchange markets (Mantegna and Stanley, 2000). A closer look of Figs 6(a)–(c) shows that as the system becomes more chaotic, the discrete spikes of the power spectrum become less evident due to increasing multiscale information transfer in the system.

The characteristic intermittency time, namely, the average duration of the laminar phases in the intermittent time series, depends on the value of the control parameter a. In the vicinity of the crisis point a_{MC} the average time spent by a path in the neighbourhood of pre-crisis CA_1 and CA_2 is very long (implying long memory), which decreases as a moves away from a_{MC} (implying shorter memory). The characteristic intermittency time (denoted by τ) can be calculated by averaging the duration of laminar phases related to CA_1/CA_2 over a long time series. Figure 7 is a plot of $\log_{10} \tau$ vs. $\log_{10}(a - a_{MC})$, where the solid line with a slope $\gamma = -0.66$ is a linear fit. The squares (circles) denote the computed average time of the laminar phases related to CA_1 (CA_2). Note that circles and squares coincide most of the time, as expected from the symmetry of CA_1 and CA_2. Figure 7 reveals that the characteristic intermittency time τ decreases with the distance from the critical parameter, obeying a power-law scaling:

$$\tau \sim (a - a_{MC})^{-0.66}. \tag{3}$$

The scaling relation for the van der Pol model of the economic type-I intermittency yields a scaling exponent of −0.074 (Chian *et al.*, 2006). Comparing with Equation 3, we see that the decrease of τ with the distance from the critical parameter for the economic crisis-induced intermittency is much faster than the economic type-I intermittency.

IV. Conclusion

Forecasting the evolution of the complex system dynamics is the ultimate goal in economics. Chaos and non-linear methods provide powerful tools to achieve this goal. For example, Bajo-Rubio *et al.* (1992) detected a chaotic behaviour on daily time series of the Spanish Peseta–US dollar exchange rate which allows short-run predictions. Soofi and

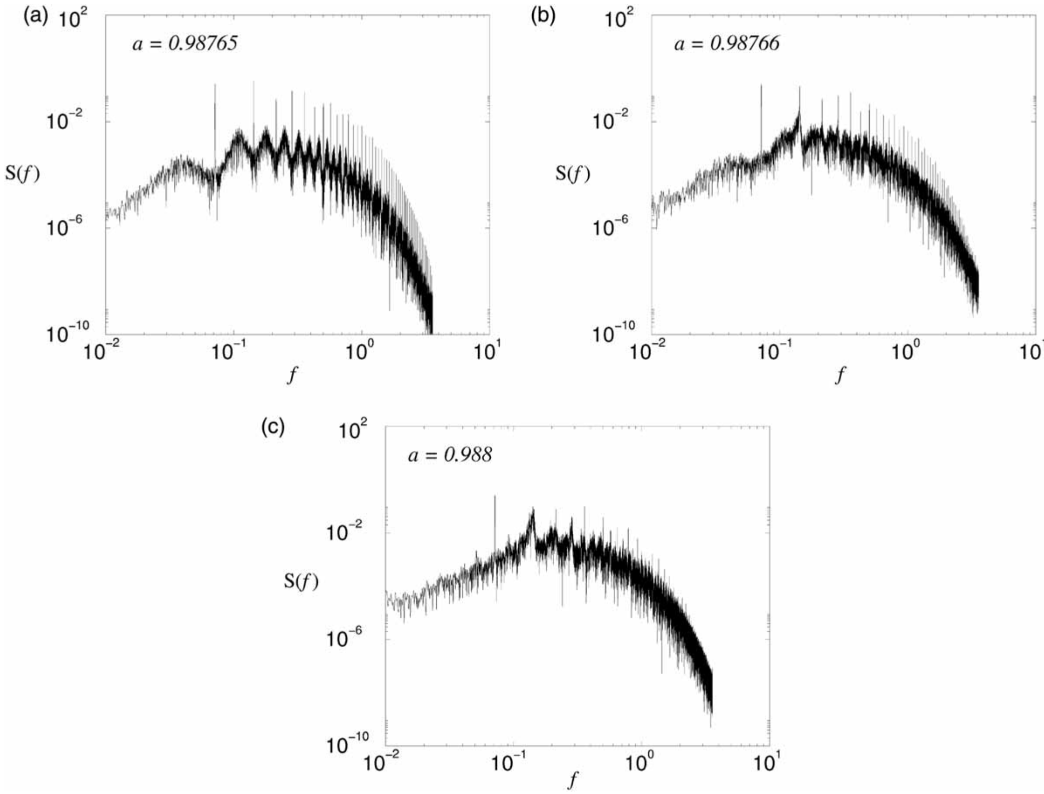

Fig. 6. Power spectrum S as a function of frequency f for: (a) $a = 0.98765$, (b) $a = 0.98766$, (c) $a = 0.988$

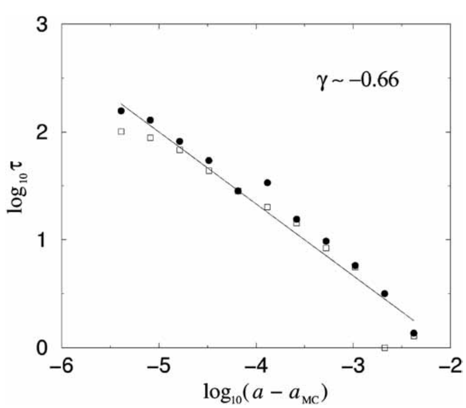

Fig. 7. Characteristic intermittency time as a function of the departure from the crisis point, $\log_{10} \tau$ vs. $\log_{10}(a - a_{MC})$. The squares (circles) denote the computed average switching time from the laminar phases related to CA_1 (CA_2) to the bursty phases. The solid line is a linear fit of the computed values with a slope $\gamma = -0.66$

Cao (1999) performed out-of-sample predictions on daily Peseta–US dollar spot exchange rates using a non-linear deterministic technique of local linear predictor. Bordignon and Lisi (2001) proposed a method to evaluate the prediction accuracy of chaotic time series by means of prediction intervals and showed its effectiveness with data generated by a chaotic economic model.

A non-linear prediction method being developed in population dynamics, weather dynamics and earthquake dynamics is based on attractor reconstruction in phase space using the time series of observed data (Drepper *et al*., 1994; Perez-Munuzuri and Gelpi, 2000; Konstantinou and Lin, 2004). This technique may be applied to economic forecasting. Information obtained from modelling intermittency of a complex economic system can guide the analysis of the reconstructed attractor by providing identifiable and predictable recurrent system patterns (Belaire-Franch, 2004), and allowing the calculation of the characteristic intermittency time for each recurrent pattern. In particular, the determination of intermittent features in the modelled economic chaotic attractors, aided by the recognition of regions of high predictability in the chaotic attractors (Ziehmann *et al*., 2000), and the calculation of the power-law scaling in the intermittent error dynamics (Chu *et al*., 2002) may reduce prediction error and improve economic forecasting precision.

Economic forecasting relies on the agent's skill to recognize the patterns of recurrence in the past economic time series and to estimate the waiting time between bursts. Recurrence of unstable periodic structures is a manifestation of the memory dynamics of complex economic systems. Dynamical systems approach provides effective tools to identify the origin and nature of the recurrent patterns. In this article, we demonstrated how economic intermittency is induced by an attractor merging crisis and how to recognize different recurrent patterns in the intermittent time series of economic cycles by separating them into laminar (weakly chaotic) and bursty (strongly chaotic) phases. The characteristic intermittency time given by the scaling relation, Equation 3, can be used to predict the turning points of regime switching from contrationary phases to expansionary phases in economic cycles.

Modelling of non-linear economic dynamics enables us to obtain an in-depth knowledge of the nature of regime switching and memory, in particular, their relation with each other. Econometric literatures on regime switching (Kirikos, 2000; Bautista, 2003; Kholodilin, 2003) and long memory (Granger and Ding, 1996; Resende and Teixeira, 2002; Gil-Alana, 2004; Muckley, 2004) have evolved largely independently, as the two phenomena appear distinct. Diebold and Inoue (2001) argued that regime switching and long memory are intimately related, which is in fact confirmed by our analysis. As an economic system evolves, microeconomic and macroeconomic instabilities lead to a variety of global and local bifurcations which in turn give rise to chaotic behaviours such as crisis-induced and type-I intermittencies. The techniques developed in this article can be applied to investigate intermittency in more complex economic models and to analyze other types of economic intermittency such as intermittency driven by a boundary crisis or an interior crisis, on-off intermittency and noise-induced intermittency.

Acknowledgements

This work is supported by CNPq and forms part of Ph.D. Thesis in Economics by A C.-L. Chian at the University of Adelaide.

References

Bajo-Rubio, O., Fernandez-Rodriguez, F. and Soscilla-Rivero, S. (1992) Chaotic behavior in exchange-rate series: first results for the Peseta-US Dollar case, *Economics Letters*, **39**, 207–11.

Bautista, C. C. (2003) Stock market volatility in Philippines, *Applied Economic Letters*, **10**, 315–8.

Belaire-Franch, J. (2004) Testing for nonlinearity in an artificial financial market: a recurrence quantification approach, *Journal of Economic Behavior & Organization*, **54**, 483–94.

Bischi, G. I., Stafanini, L. and Gardini, L. (1998) Synchronization, intermittency and critical curves in a duopoly game, *Mathematics and Computers in Simulation*, **44**, 559–85.

Bordignon, S. and Lisi, F. (2001) Predictive accuracy for chaotic economic models, *Economics Letters*, **70**, 51–8.

Chian, A. C.-L., Borotto, F. A., Rempel, E. L. and Rogers, C. (2005) Attractor merging crisis in chaotic business cycles, *Chaos, Solitons and Fractals*, **24**, 869–75.

Chian, A. C.-L., Rempel, E. L., Borotto, F. A. and Rogers C. (2006) An example of intermittency in nonlinear economic cycles, *Applied Economic Letters*, **13**, 257–63.

Chu, P. C., Ivanov, L. M., Kantha, L. H., Melnishenko, O. V. and Poberezhny, Y. A. (2002) Power law decay in model predictability skill, *Geophysical Research Letters*, **29**, Art. No. 1748.

Diebold, F. X. and Inoue, A. (2001) Long momory and regime switching, *Journal of Econometrics*, **105**, 131–59.

Drepper, F. R., Engbert, R. and Stollenwerk, N. (1994) Nonlinear time series analysis of empirical population dynamics, *Ecological Modelling*, **75**, 171–81.

Gil-Alana, L. A. (2004) The dynamics of the real exchange rates in Europe: a comparative study across countries using fractional integration, *Applied Economic Letters*, **11**, 429–32.

Granger, C. W. J. and Ding, Z. (1996) Varieties of long memory models, *Journal of Econometrics*, **73**, 61–77.

Haxholdt, C., Kampmann, C., Mosekilde, E. and Sterman, J. D. (1995) Mode-locking and entrainment of endogenous economic cycles, *System Dynamics Review*, **11**, 177–98.

Kholodilin, K. A. (2003) US composite economic indicator with nonlinear dynamics and the data subject to structure breaks, *Applied Economic Letters*, **10**, 363–72.

Kirikos, D. G. (2000) Forecasting exchange rates out of sample: random walk vs. Markov switching regimes, *Applied Economic Letters*, **7**, 133–6.

Konstantinou, K. I. and Lin, C. H. (2004) Nonlinear time series analysis of tremor events recorded at Sangay volcano, Ecuador, *Pure and Applied Geo-physics*, **161**, 145–63.

Mantegna, R. N. and Stanley, H. E. (2000) *An Introduction to Econophysics: Correlations and Complexity in Finance*, Cambridge University Press, Cambridge.

Mosekilde, E., Larsen, E. R., Sterman, J. D. and Thomse, J. S. (1992) Nonlinear mode-interaction in the macroeconomy, *Annals of Operations Research*, **37**, 185–215.

Muckley, C. (2004) Empirical asset return distributions: is chaos the culprit?, *Applied Economic Letters*, **11**, 81–6.

Mullineux, A. W. (1990) *Business Cycles and Financial Crisis*, Harvester Wheat-sheaf, Hemel Hemstread.

Perez-Munuzuri, V. and Gelpi, I. R. (2000) Application of nonlinear forecasting techniques for meteorological modeling, *Annales Geophysicae*, **18**, 1349–59.

Resende, M. and Teixeira, N. (2002) Permanent structural change in the Brazilian economy and long momory: a stock market perspective, *Applied Economic Letters*, **9**, 373–75.

Soofi, A. S. and Cao, L. (1999) Nonlinear deterministic forecasting of daily PesetaDollar exchange rate, *Economics Letters*, **62**, 175–80.

Ziehmann, C., Smith, L. A. and Kurths, J. (2000) Localized Lyapunov exponents and the prediction of predictability, *Physics Letters A*, **271**, 237–51.