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Characterisation of deep bed filtration system from laboratory pressure drop measurements

P. Bedrikovetsky [a,*], D. Marchesin [b], F. Shecaira [c], A.L. Souza [c], P.V. Milanez [c], E. Rezende [a]

[a] State University of Norte Fluminense LENEP/UENF, Rod. Amaral Peixoto 164, 27973-030 Macaé, RJ, Brazil
[b] Instituto de Matemática Pura e Aplicada IMPA, Est. Dona Castorina 110, 22460-320 Rio de Janeiro, Brazil
[c] Petrobras Research Centre CENPES, Cid. Universitária-q.7, Ilha do Fundão, 21949-900 Rio de Janeiro, RJ, Brazil

Abstract

During the injection of sea/produced water, permeability decline occurs, resulting in well impairment. Solid and liquid particles dispersed in the injected water are trapped by the porous medium and may increase significantly the hydraulic resistance to the flow. We formulate a mathematical model for deep bed filtration containing two empirical parameters—the filtration coefficient and the formation damage coefficient. These parameters should be determined from laboratory coreflood tests by forcing water with particles to flow through core samples. A routine laboratory method determines the filtration coefficient from expensive and difficult particle concentration measurements of the core effluent; then, the formation damage coefficient is determined from inexpensive and simple pressure drop measurements. An alternative method would be to use solely pressure difference between the core ends. However, we prove that given pressure drop data in seawater coreflood laboratory experiments, solving for the filtration and formation damage coefficients is an inverse problem that determines only a combination of these two parameters, rather than each of them. Despite this limitation, we show how to recover useful information on the range of the parameters using this method. We propose a new method for the simultaneous determination of both coefficients. The new feature of the method is that it uses pressure data at an intermediate point of the core, supplementing pressure measurements at the core inlet and outlet. The proposed method furnishes unique values for the two coefficients, and the solution is stable with respect to small perturbations of the pressure data. © 2001 Published by Elsevier Science B.V.

Keywords: Deep bed filtration; Formation damage; Inverse problem; Sea water injection; Well impairment

1. Introduction

Injectivity decline of oilfield injection wells is a widespread phenomenon during sea/produced water injection. This decline may result in significant cost increase in the waterflooding project. Reliable prediction of this decline is important for waterflood design as well as for choice and preventive treatment of the injected water (Todd et al., 1979; Rochon and Creosot, 1996). One of the reasons for well injectivity decline is permeability decrease due to rock matrix plugging by solid/liquid particles suspended in the injected water (the flow and deposition of particles in the rock matrix is called deep bed filtration).

The mathematical model for deep bed filtration presented by Herzig et al. (1970) and by Sharma and Yortsos (1987) contains two empirical parameters—the filtration coefficient λ and the formation damage

* Corresponding author.
E-mail addresses: pavel@lenep.uenf.br (P. Bedrikovetsky), marchesi@impa.br (D. Marchesin).

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coefficient β. Knowledge of these two parameters is essential for predicting well injectivity decline during sea/produced water injection. These parameters are empirical; therefore, they should be determined from laboratory coreflood tests by flowing water with particles through the rock.

Pang and Sharma (1994) and Wennberg and Sharma (1997) showed that both parameters can be inferred from combined measurements of core pressure drop and of suspended particle concentration in the core outlet water.

Usually, a coreflood test is accompanied by pressure drop measurements. These measurements are inexpensive and simple to perform and, therefore, they are widespread in the literature. Nevertheless, suspended particle concentration data in core outlet water during laboratory tests are almost unavailable in the literature. This is so because the measurement of concentration data requires special equipment and it is difficult as compared to pressure drop measurements. (Pang and Sharma, 1994; Oort et al., 1993; Soo et al., 1986; Wennberg and Sharma, 1997; Bedrikovetsky et al., 2000, 2001).

These difficulties are the motivation for attempting to determine the constants λ and β from total pressure drop along the core measured at different times during the flow. Here, we show that the mathematical solution of this problem has limitations. This is discussed in Section 2 heuristically and in Section 3 rigorously. In summary, only a combination of these two parameters can be found. At best, only ranges of each of these two parameters can be obtained. The procedure to obtain such ranges is shown through an example in Section 3.

This limited result is unfortunate for common engineering practice. For example, certain existing software packages for predicting well injectivity loss provide the option of adjusting the pressure drop curve by matching both parameters λ and β, under the implicit assumption that these two parameters can be found from the test.

In the present work, we propose a method for determining the filtration and formation damage coefficients from pressure measurements at an intermediate point of the core as well as at core entrance and exit during deep bed filtration. Section 4 describes the laboratory procedure and Section 5 the mathematical recovery method. It is proven in Section 6 that the method furnishes unique values for the two coefficients, and that the solution of the inverse problem is unique and stable. Another example is discussed in Section 8. The precise mathematical description of our model is contained in Appendix A.

2. Empirical coefficients in the model for deep bed filtration

Well impairment during injected sea water flow in porous media occurs due to solid/liquid particle entrapment by the pore throats and by the matrix. Deep bed filtration is modelled by a system containing equations for particle mass balance in dispersed and captured states (Eq. (A-1)), for particle capture rate (Eq. (A-2)), as well as a modified version of Darcy's law taking into account formation damage (Eq. (A-3)) with functional expression $k(\sigma)$ (given for example in Eq. (A-4)).

Particle entrapment occurs due to a variety of physical mechanisms like interception, bridging, sorption, sedimentation, attraction by molecular forces (Payatakes et al., 1974; Roque et al., 1995). Nevertheless, a single equation of linear kinetics (Eq. (A-2)) with an empirical filtration function $\lambda(\sigma)$ is used to describe the capturing process due to all different mechanisms.

Following the works of Pang and Sharma (1994), Wennberg and Sharma (1997) and Bedrikovetsky et. al., 2000, 2001, we consider a simplified model with a constant filtration coefficient λ = constant, and with a hyperbolic functional expression for the formation damage dependence (Eq. (A-4)). The system of governing Eqs. (A-1)–(A-3) in dimensionless coordinates (Eq. (A-5)) has the form of (Eqs. (A-6)–(A-8)). The system contains two dimensionless parameters—β and λL. The magnitude of the porous media impairment is determined by the rate of particle capture, represented by the value of the filtration coefficient λ and by the permeability decline, represented by the value of the formation damage constant β. These are the two empirical parameters to be determined from laboratory data.

This linear one-dimensional flow model (with initial and boundary conditions given by Eqs. (A-9) and (A-10), respectively) corresponds to a laboratory coreflood test of water injection with particle concentration c_m into a core previously filled with sweet water (Fig. 1). This problem has a closed solution (Herzig et al., 1970; Pang and Sharma, 1994). The concentration front $X(T) = T$ moves through the core with constant velocity equal to the flow velocity; ahead of this front both dispersed and deposited particle concentrations $c(X,T)$ and $\sigma(X,T)$ vanish. The distributions of dispersed and deposited particle concentrations behind the front are given by Eqs. (A-11) and (A-12), respectively.

The dimensionless pressure drop (Eq. (A-13)), which we call impedance index, reflects the relative decline of the average core permeability during deep bed filtration. By substituting the expression for deposited concentration into Darcy's law (Eq. (A-8)) and integrating the pressure gradient in X, we obtain the expression for impedance index growth. At time $T = 1$, when the concentration front arrives at the core outlet, a dispersed steady state concentration (Eq. (A-12)) has been established. Integrating in X from zero to one yields the impedance index expression (Eq. (A-14)) for $T > 1$.

Before the front arrival at the core end ($T < 1$), the dispersed concentration is in steady state only behind the front, and it vanishes ahead of the front. Integration yields the impedance index expression (Eq. (A-15)) for $T < 1$.

Eq. (A-14) shows that the plot $J(T)$ for $T > 1$ is a straight line. Fig. 2 shows results from a laboratory coreflood test involving sea water containing solid particles with concentration $c_m = 0.85$ ppm (parts per million) in a core with initial permeability $k_0 = 0.153 \times 10^{-12}$ m^2, porosity $\phi = 0.24$, and core length $L = 0.1$ m. The core and the injected water were taken from a certain Brazilian giant deep water offshore

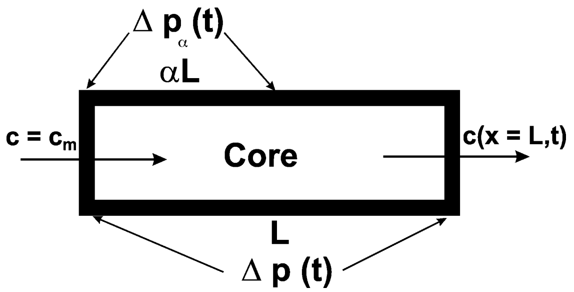

Fig. 1. Schematic diagram of pressure measurements during coreflood test with sea/produced water flow.

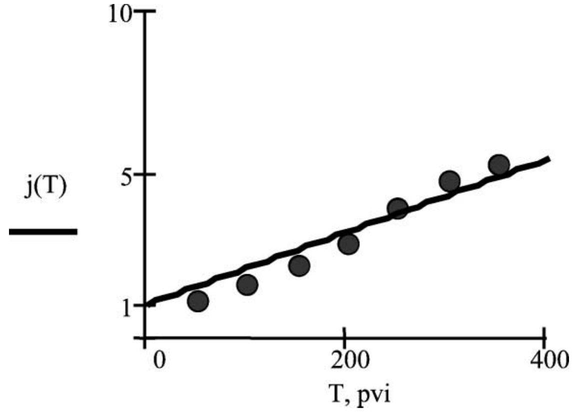

Fig. 2. Impedance variation during coreflood test (laboratory data) for $T > 1$.

field, submitted to sea waterflooding and suffering from significant injector well impairment. The points in Fig. 2 present measurements at seven different times in the range of 50–350 pvi (porous volumes injected). The straight line $J(T)$ is obtained by the least square method.

The slope m of the impedance line $J(T)$ is determined by the values of the filtration coefficient λ and of the formation damage coefficient β, (Eq. (A-16)). So, the value m as determined from laboratory data $J(T)$ defines the curve $\beta = \beta(\lambda)$ on the plane (λ,β) given by Eq. (A-17). In the above-mentioned laboratory test, the slope is 0.011. The curve $\beta = \beta(\lambda)$ in

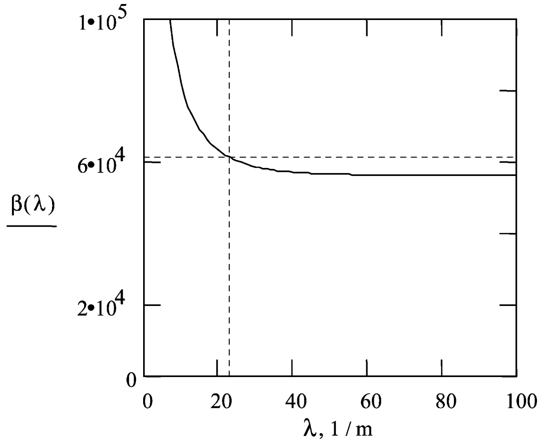

Fig. 3. Dependence of formation damage coefficient versus filtration coefficient as obtained from impedance variation.

Fig. 3 is obtained from the laboratory test data as shown in Fig. 2.

The function $\beta=\beta(\lambda)$ decreases monotonically. Thus, the effects of the formation damage β and of the capturing intensity λ compensate each other—the same impedance growth m can be achieved by slow deposition with large permeability decline, or by fast deposition with small permeability decrease. The function $\beta=\beta(\lambda)$ tends to infinity when λ tends to zero (Fig. 3), as well as a horizontal asymptote $\beta=m/(c_m\phi)$ when λ tends to infinity.

The impedance index curve $J(T)$ given by Eq. (A-15) for $T<1$ is non-linear. The curve $J(T)$ is concave, its derivative increases steadily from zero at $T=0$ to m at $T=1$, reflecting formation damage increase in the zone $X\leq T$ and deposited particle accumulation. The impedance index curve $J(T)$ for $T<1$ is shown in Fig. 4.

The filtration coefficient can be determined from dispersed particle concentration measurements at the core outlet $c(X=1,T)$ after the front arrival ($T>1$) (Pang and Sharma, 1994; Wennberg and Sharma, 1997). An explicit formula for λ follows from Eq. (A-12). Then, the formation damage coefficient β can be determined from the pressure drop data.

In the above laboratory test, the outlet concentration has also been measured. For $c(X=1,T)=0.092$ ppm, the value $\lambda=22.3$ 1/m is obtained. Finding β for $\lambda=22.3$ on the curve $\beta=\beta(\lambda)$ in Fig. 3 yields the value $\beta=6.17\times10^4$.

The methodology for deep bed filtration characterisation from pressure drop data and outlet concentration can be generalised for non-linear deep bed filtration systems with general filtration and damage functions $\lambda(\sigma)$ or $k(\sigma)$. The problem of finding $\lambda(\sigma)$ from the outlet concentration $c(X=1,T)$ yields a functional equation with a unique and stable solution (Bedrikovetsky et al., 2001). The problem of finding $k(\sigma)$ from the pressure drop $\Delta p(T)$ between the core inlet and outlet yields an integral equation that has also a unique and stable solution (Bedrikovetsky, 1993). Both inverse problems are well posed.

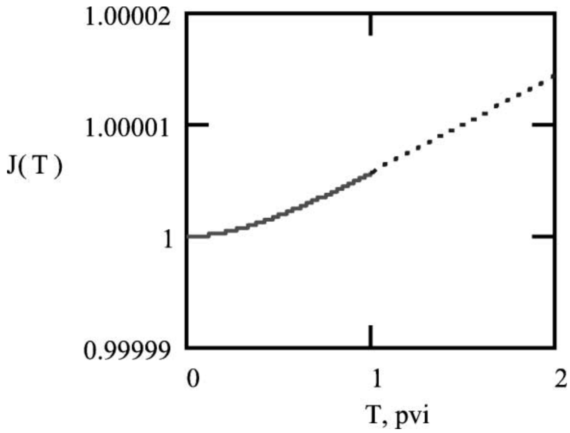

Fig. 4. Impedance variation during coreflood test for $T<1$.

3. Treatment of pressure drop data

The impedance index J as a function of time T given in Eq. (A-14) has a straight-line graph for $T>1$. Each straight line is determined by a slope and a value at $T=0$. Formula (A-16) shows that the slope and the value are functions of λ and β. One would think that these two quantities can determine the two independent parameters λ and β. (Thus, these quantities would serve as basis for software with an option to adjust a curve $J(T)$ by matching the parameters λ and β independently.) Now, we will show that this is not quite possible.

3.1. Limitations of parameter recovery

We rewrite the impedance index Eq. (A-13) in the following form:

$$J(T)=1+\beta\phi c_m(1-\exp(-x))T + \beta\phi c_m y(x),\ \text{where}\quad x=\lambda L,\ \text{and} \tag{1}$$

$$y(x)=\exp(-x)+\frac{\exp(-x)-1}{x} \tag{2}$$

Let us estimate the last term containing $y(x)$ in Eq. (1); our aim is to show that it is negligible in practice. Fig. 5 shows the plot of the function $y(x)$. The function $y(x)$ is negative for $x>0$, it vanishes at $x=0$, it tends to zero as x tends to infinity, and it has a minimum value at $x\cong1.79$. Usually, the filtration coefficient λ varies from 10 to 10^3 m^{-1}. The values of λL vary in the interval $[1,10^2]$ and cover the minimum point. Therefore, in order to evaluate the minimum value of expression (2), we find the minimum at $x=\lambda L=1.79$, which is $y(1.79)=-0.29843$. It corresponds to $J(0)=1-1.19\times10^{-3}$.

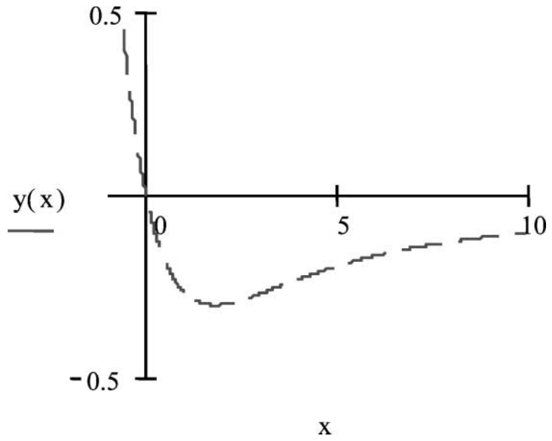

Fig. 5. Determination of maximum deviation from one of impedance $J(0)$.

If a steady state in the core fluid were established instantly, rather than after time $T=1$, the straight line $J(T)$ would start exactly at the point 1, that is $J(0)=1$. The small parameter δ in the formula for impedance index (Eq. (A-16)) is a heritage of the concentration unsteady state period. However, the system for $T>1$ forgets quickly the transition period $T<1$. Thus, the straight line (Eq. (1)) crosses the axis $T=0$ at a point very close to (0,1). These facts follow from Eqs. (A-11) and (A-12). Therefore, only the slope m of the straight line (Eq. (1)) varies substantially from test to test, and the two parameters λ and β cannot be determined independently. The value m determines only the dependence $\beta=\beta(\lambda)$ given through the formula $\beta(\lambda)=m/(c_m\phi(1-e^{-\lambda L}))$ (Fig. 2).

Plotting the impedance corresponding to different points of the curve $\beta=\beta(\lambda)$, we see that only the value δ in Eq. (A-16) is affected, rendering impossible to tell apart different lines $J(T)$ in the scale of Fig. 2. This concludes our reasoning that only $\beta(\lambda)$ can be determined in practice.

3.2. Recovery of parameter ranges

We will show now that quite a lot of information can be obtained from $\beta(\lambda)$. Let us determine the range of the slope m:

$$m=\beta\phi c_m(1-\exp(-x)), \qquad x=\lambda L \tag{3}$$

Eq. (3) shows that m is a monotonic function in β and $x=\lambda L$, such that extrema of m correspond to extrema of each parameter: if the particle concentration is low, $c_m=0.2\times10^{-6}$ for $\beta=10^3$ and $\lambda L=10^2$, we obtain $m=4\times10^{-5}$; for $\beta=10$ and $\lambda L=1$, we obtain $m=2.5\times10^{-7}$; if the particle concentration is high, $c_{\mathrm{m}}=20\times10^{-6}$ for $\beta=10^3$ and $\lambda L=10^2$, we obtain $m=4\times10^{-3}$; for $\beta=10$ and $\lambda L=1$, we obtain $m=2.5\times10^{-5}$. So, variation of the independent parameters λ and β causes slope changes of two orders of magnitude.

If the parameter β is fixed, the change is significantly smaller: when λ varies from 10^1 to 10^3, the term $1-\exp(-\lambda L)$ in Eq. (4) varies from 2.5 to 4.0. For different values of c_m and β, the absolute variation can be significantly higher, but the relative variation is the same: 2.5–4.0. The plot $m=m(\lambda)$ shown for the typical value $L=0.1$ m indicates that the variation of λ lies in the range [10,50]. It follows that values of $\lambda>50$ cannot be found with confidence. The explanation of this phenomenon is the following: the slope m is a linear function of β, but it tends to a finite limit when $\lambda\to\infty$. Therefore, for a fixed λ, the m-data determines β for any m-value. Nevertheless, for a fixed β, the m-data determines λ only for m-values with λ in the range [10,50]. For $\lambda>50$, the solution of λ from the m-data is not possible.

It is interesting to point out that while the "overall formation damage in the core" m and the filtration coefficient increase monotonically together, the deposited concentration σ depends non-monotonically on the filtration coefficient. The deposited concentration $\sigma(X,T)$ increases monotonically from zero to its maximum value when λL increases from zero to $1/X$, and then it decreases to zero when λL tends to infinity. The reason for this non-monotonicity is that the filtration coefficient has a two-way effect on the deposited concentration σ: a higher filtration coefficient increases the first term in the expression for the deposition rate (Eq. (A-2)) but it lowers the water particle concentration c in Eq. (A-1). The product of these two variables is proportional to the deposition rate (Eq. (A-2)), hence σ depends non-monotonically on λL.

The exact expression (Eq. (3)) for m allows estimating the formation damage coefficient β. Let $T_{1/2}$ be the time when overall core permeability halves. As it

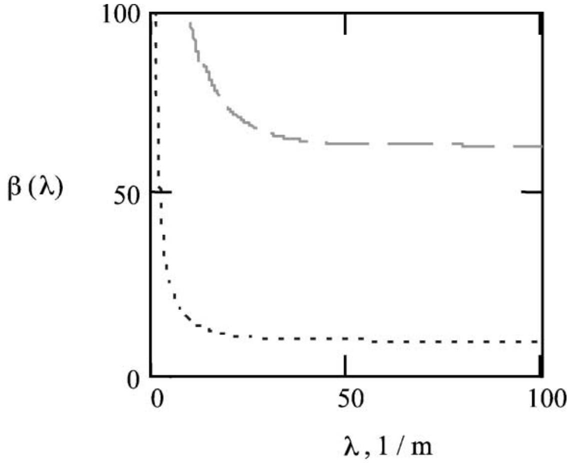

Fig. 6. Curves $\beta(\lambda)$ for different values of impedance slope m.

follows from Eq. (3), $m=1/T_{1/2}$. Let us substitute m into Eq. (3):

$$\beta(1-\exp(-\lambda L))=\frac{1}{T_{1/2}c_m\phi} \tag{4}$$

The $e^{-\lambda L}$ term on the left-hand side of Eq. (4) varies from zero to one. Therefore

$$\beta>\frac{1}{T_{1/2}c_m\phi} \tag{5}$$

For the data set $T_{1/2}=10^2,\ 10^3,\ 10^4$, $c_m=0.1\times10^{-6},\ 10^{-6},\ 10\times10^{-6}$ and $\phi=0.1$, we obtain the following estimates for β: 10^6, 10^5, 10^4, 10^3 and 10^2. It seems that β should always be larger than 10^2.

Fig. 6 illustrates the indeterminacy of the filtration and damage coefficients based on pressure drop data. Curves 1, 2 and 3 correspond to different values of the slope m. One sees that for a given slope value, values of λ can be chosen lying in whole ranges. The value of β that matches laboratory data is uniquely determined for each choice of a point in the curve in Fig. 6.

4. Description of a two-pressure-drop test

We have established that only a relationship $\beta(\lambda)$ can be determined from a simple and inexpensive pressure drop measurement on coreflood using sea/produced water. In order to determine the parameters β and λ separately, one could use as data the particle concentration at the core outlet. However, concentration measurements are difficult to perform accurately and require expensive equipment. We propose the measurement of the pressure at an intermediate core point and its usage as an independent data source for the determination of the parameters β and λ separately.

The laboratory test scheme is shown in Fig. 1. During the particulate water injection, the pressures $p(x=0,\ t)$, $p(x=\alpha L,\ t)$ and $p(x=L,\ t)$ are measured. Here, αL (with $0<\alpha<1$) is an intermediate point the core. The flow rate variation $U(t)$ is also measured during the test.

The initial permeability is determined from the pressure drop at the beginning of the flood, where the rock is not impaired yet. The length of the core L and the position of the intermediate point αL are known. Of course, both the porosity of the core and the viscosity of the injected water have been determined before the flood.

The problem solved in the next section is the determination of the parameters λ and β from these measurements.

5. Impedance calculations in two core zones

The slope m of the impedance index line is given by Eq. (A-16):

$$m=\beta c_m\phi(1-e^{-\lambda L}) \tag{6}$$

The corresponding curve $\beta(\lambda)$ is shown in Fig. 7 as a solid curve.

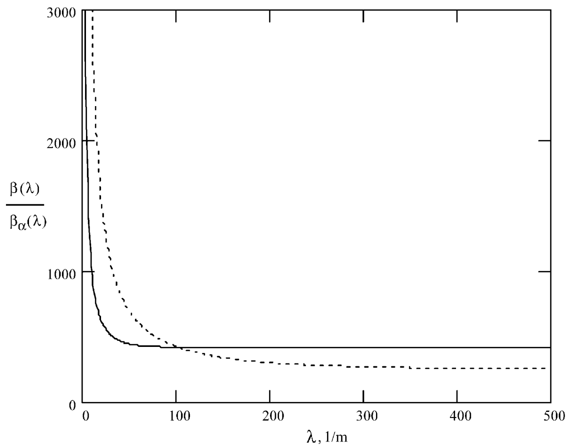

Fig. 7. Graphical determination of coefficients β and λ from data on two pressure drops.

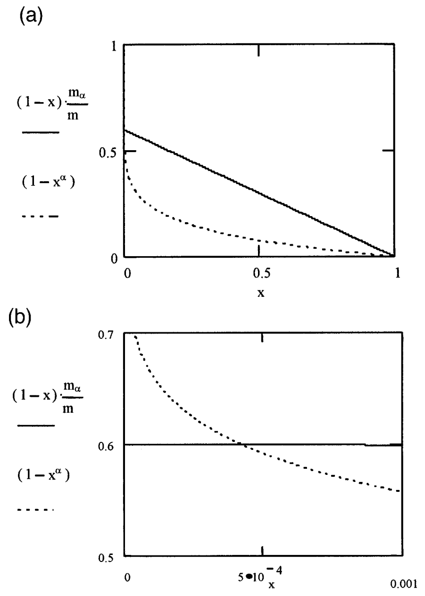

Fig. 8. Unique determination of coefficients β and λ from data on two pressure drops: (a) for the range $0<x<1$; (b) for the range $0<x<0.001$.

Let us discuss the slope m_α as obtained from the pressure drop on part of the core. The length of the core L in Eq. (6) should be replaced by αL. The formula for m_α becomes:

$$m_\alpha = \beta c_m \phi (1 - e^{-\lambda \alpha L}) \tag{7}$$

Eq. (7) also determines the curve $\beta = \beta(\lambda)$ on the plane (β, λ), shown in Fig. 7 as a dashed curve.

Eqs. (6) and (7) form a system of two equations with unknowns β and λ. The solution corresponds to the intersection point of the two curves (Fig. 7). Let us divide Eq. (6) by Eq. (7) to obtain:

$$(1-y)\frac{m_\alpha}{m} = (1-y^\alpha) \tag{8}$$

$$\lambda = -\frac{1}{L}\ln(y) \tag{9}$$

Eq. (8) can be used to determine y by solving this transcendental equation numerically. Let us show that this equation has always a solution, and this solution is unique, provided $\alpha > (m_\alpha/m)$. This condition is equivalent to $\lambda > 0$, that is, that there is indeed deposition.

Fig. 8a presents plots of the left and right hand sides of Eq. (8) for $0<\alpha<1$. As one can see from Eq. (8), the straight line on the left hand side of this equation starts at the point $(0, m_\alpha/m)$ and ends at (1,0), and has slope m_α/m. The plot of the right-hand side curve is concave; this curve starts at the point (0,1) and ends at (1,0). Its slope at the latter point is α. Therefore, the two curves have at most a unique intersection, besides the point (0,1). Thus, the values of β and λ can be uniquely determined from the data m and m_α. For the example presented in Fig. 8a, the value of the root is 4.6×10^{-4}, so the intersection point is only visible in the zoomed plot (Fig. 8b).

For $\alpha = 1/2$, the intermediate point is located at the middle of the core, and the solution of Eq. (8) has a simple explicit expression, which provides an answer $y<1$ provided $m_\alpha < (m/2)$:

$$y = \left(\frac{m - m_\alpha}{m_\alpha}\right)^2 \tag{10}$$

For $\alpha = 1/3$ and $\alpha = 1/4$, more complicated explicit solutions can also be found.

Fig. 9 presents the analysis of coreflood data taken from the papers by Tran and van den Broek (1998) and van den Broek et al. (1999). Fig. 9 shows two plots: the straight-line impedance index versus core pore volumes injected, and the impedance index for the α-th part of the core. One can see that $m_\alpha < m$. The difference in the slopes m_α and m is due to the fact that formation damage is stronger near the inlet than on the rest of the core.

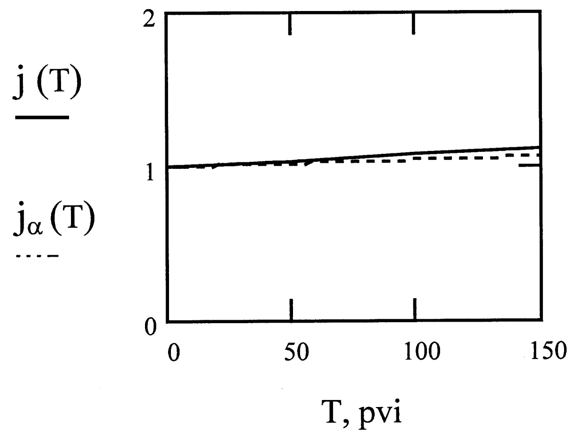

Fig. 9. Different impedance slopes obtained from pressure drops on the whole core length and on part of the core length.

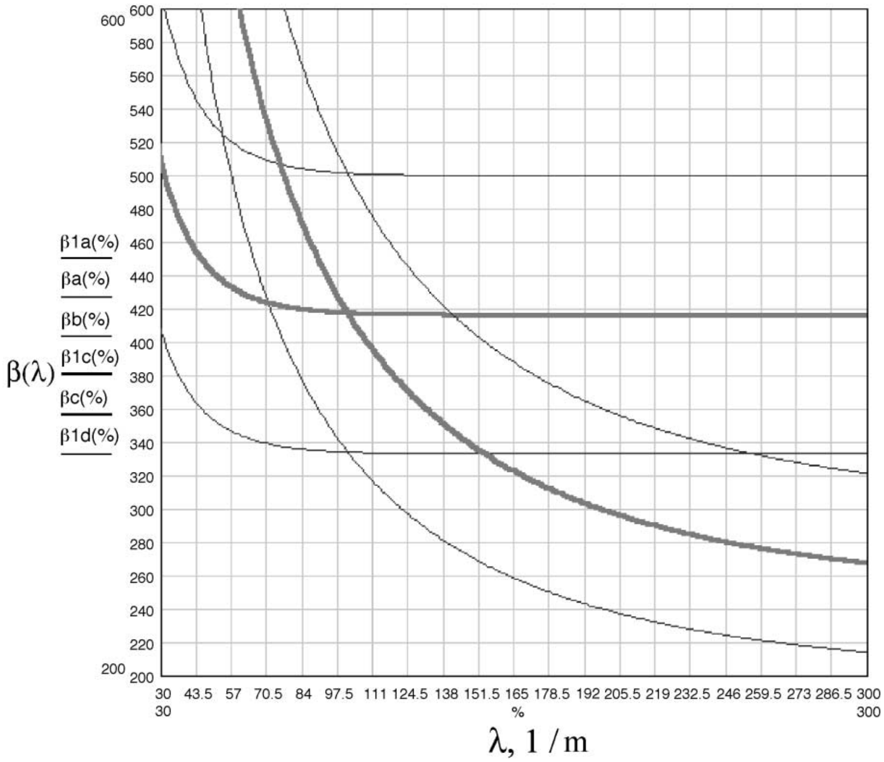

Fig. 10. Stability of results with regard to small perturbation of laboratory data.

Let us apply the method above to some experimental data. Pressure measurements have been taken at three points of a Berea core; in this case, $\alpha = 0.118$. The slope values have been calculated from pressure drop measurements: $m = 7.5 \times 10^{-4}$, $m_\alpha = 4.5 \times 10^{-4}$. The corresponding curves $\beta = \beta(\lambda)$ are given in Fig. 7. The intersection point determines the values of filtration and formation damage coefficients: $\beta = 417.9$, $\lambda = 101.3$.

6. Sensitivity analysis of the results

The slopes m and m_α are determined from impedance index curves, which are calculated from pressure drop data determined in the laboratory test. Because these measurements are made with limited accuracy, it is important to check whether the method proposed yields stable results, i.e. whether small variations in the quantities m and m_α yield small variations in the coefficients β and λ.

Fig. 10 shows four cases with values of m and m_α perturbed by $\pm 20\%$. The values of m and m_α are the same as obtained in the above-mentioned test (Fig. 7).

The pairs of curves resulting from m and m_α data are shown in Fig. 10 for the four 'perturbed' points. The 'perturbed' pairs are shown as thin curves, and the 'unperturbed' curves already obtained in Fig. 7 are shown as thick curves. The 'perturbed' points β and λ are located very near the 'unperturbed' point $\beta = 417.9$, $\lambda = 101.3$. This is evidence for stability of the procedure.

7. Treatment of other sets of laboratory test data

In another test by Tran and van den Broek (1998) and van den Broek et al. (1999), the Bentheimer core was flooded by water with solid particles. The setup for this test was the same as that of Fig. 7. The slope values are calculated from pressure drop measurements, yielding $m = 2.5 \times 10^{-4}$, $m_\alpha = 1.1 \times 10^{-4}$,

$\beta(\lambda)$ and $\beta_\alpha(\lambda)$. The intersection of the two curves $\beta(\lambda)$ is the point $\beta=394$, $\lambda=61.5$ 1/m.

Another Berea core was flooded by oily water (van den Broek et al., 1999). The slope values are: $m=1.9\times10^{-4}$, $m_1=4.6\times10^{-5}$. The intersection point is $\beta=17$, $\lambda=20$ 1/m.

8. Conclusions

The analysis of the solution of the deep bed filtration characterisation problem from pressure drop data during linear flow of water with particles in porous medium yields the following conclusions:

1. The straight-line 'impedance index versus time, pvi', as calculated after injection of one pore volume intersects the ordinate axis $T=0$ at a point very close to (0,1).
2. Only the slope m of the straight line $J(T)$ is an independent parameter that can be determined reliably from pressure drop data during coreflood experiments.
3. The value of m determines a relationship $m(\lambda,\beta)=\text{constant}$ between the parameters λ and β. These parameters cannot be determined separately based on pressure drop measurements during coreflood by particulate water.
4. A method to determine the filtration and formation damage coefficients β and λ from pressure measurements at the core inlet and outlet and at an intermediate point is proposed.
5. The values of these two coefficients β and λ are given by a unique solution of the inverse problem from pressure data at three points on the core.
6. The unique solution is stable with respect to small perturbations of measured pressure drops.

Nomenclature

t	time
T	dimensionless time in pore volumes
x	linear distance
X	dimensionless distance
L	length of the core
αL	position of the intermediate point for pressure measurement
c	suspended particles concentration
c_m	suspended particles concentration in the injected water
σ	deposited particles concentration
k_0	original (formation/core) permeability before injection
k	permeability as a function of σ, normalized by k_0
J	impedance
m	slope of a linear plot $J(T)$
m_α	slope of the plot $J(T)$ for the α-th part of the core
μ	water viscosity
λ	filtration coefficient
β	formation damage coefficient
U	flow velocity
p	pressure
ϕ	porosity

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Appendix A. Determination of the relationship $\beta(\lambda)$ from pressure drop data

The deep bed filtration process is described by equations of mass balance for particles in dispersed and trapped states, of entrapment kinetics and of a modified form of Darcy's law accounting for the permeability dependence on deposited concentration:

$$\phi\frac{\partial c}{\partial t}+U\frac{\partial c}{\partial x}=-\frac{\partial\sigma}{\partial t} \qquad (A-1)$$

$$\frac{\partial \sigma}{\partial t} = \lambda U c \qquad (A-2)$$

$$U = -\frac{k_0 k(\sigma)}{\mu} \frac{\partial p}{\partial x}. \qquad (A-3)$$

Here a hyperbolic form is assumed for the formation damage function $k(\sigma)$:

$$k(\sigma) = \frac{1}{1+\beta\sigma}. \qquad (A-4)$$

Let us introduce dimensionless length and time:

$$X = \frac{x}{L}; \quad T = \frac{Ut}{\Phi L}. \qquad (A-5)$$

The system of governing (Eqs. (A-1)–(A-3)) takes the following form in dimensionless coordinates:

$$\frac{\partial c}{\partial T} + \frac{\partial c}{\partial X} = -\lambda L c \qquad (A-6)$$

$$\frac{\partial \sigma}{\partial T} = \lambda L \phi c \qquad (A-7)$$

$$U = -\frac{k_0 k(\sigma)}{\mu} \frac{\partial p}{\partial X} \qquad (A-8)$$

The initial condition for the systems (A-6)–(A-8) corresponds to absence of particles in the porous medium water before injection:

$$T = 0 \ : \quad c = 0\ ; \quad \sigma = 0 \qquad (A-9)$$

The boundary condition for the systems (A-6)–(A-8) corresponds to water injection with a specified solid particle concentration

$$X = 0 \ : \quad c = c_m \qquad (A-10)$$

If the filtration coefficient λ is constant, the solution of (Eqs. (A-6), (A-7), (A-9), (A-10)) for $X < T$ is:

$$\sigma(X,T) = \lambda\ L\phi c_m (T - X)\exp(-\lambda L X) \qquad (A-11)$$

$$c(X,T) = c_m \exp(-\lambda L X) \qquad (A-12)$$

For $X > T$, ahead of the front $X = T$, both concentrations c and σ vanish.

So, for $T > 1$, the front has already reached the core outlet, and steady state concentration distribution c has already been established. For $T < 1$, the concentration distribution c is unsteady. The deposition distribution σ is never steady.

Integrating in X the pressure gradient from Eq. (A-8) within $0 < X < 1$, and substituting the solution (A-11) into the result, we obtain an expression for the impedance index J (which is the inverse of injectivity index) valid for $T > 1$:

$$J(T) = \frac{k_0}{k(T)} = \frac{k_0 \Delta p(T)}{\mu L U(T)}; \qquad (A-13)$$

$$J(T) = 1 + \beta\phi c_m \left(T(1 - \exp(-\lambda L)) + \exp(-\lambda L) + \frac{\exp(-\lambda L) - 1}{\lambda L} \right) \qquad (A-14)$$

Integrating in X the pressure gradient from Eq. (A-8) within $0 < X < T$, and substituting the solution (A-11) into the result yields an expression for the impedance function J valid for $T < 1$:

$$J(T) = 1 + \beta\phi c_m \left(T(1 - \exp(-\lambda L T)) + T\exp(-\lambda L T) + \frac{\exp(-\lambda L T) - 1}{\lambda L} \right) \qquad (A-15)$$

Thus, deep bed filtration injectivity decline depends on two dimensionless parameters: λL and $\beta\phi c_m$.

According to Eq. (A-14), the curve $J(T)$ is a straight line for $T > 1$.

$$J(T) = mT + 1 - \delta \qquad (A-16)$$

$$m = \beta c_m \phi (1 - e^{-\lambda L}) \qquad (A-17)$$

$$\delta = -\beta c_m \phi \left(e^{-\lambda L} + \frac{1 - e^{-\lambda L}}{\lambda L} \right) \qquad (A-18)$$

The parameter δ in Eq. (A-16) has the order of magnitude 10^{-4}, therefore, only the slope m can be determined from the plot $J = J(T)$, as discussed in Section 3.1.

Thus, the simplified Eq. (A-16), with $\delta = 0$ and just one unknown parameter m, is used to determine impedance index growth.

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