DISPLACEMENT OF OIL BY A SLUG OF AN ACTIVE ADDITIVE FORCED BY WATER THROUGH A STRATUM

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The paper gives a solution to the problem of the displacement of oil by a slug for different forms of the sorption isotherm and the distribution function of the additive between the phases and for different values of the initial flooding of the stratum. The process is considered under conditions of reversible sorption and also under conditions of partial retention of the additive by the skeleton of the porous medium. The behavior of slugs in the case of cyclic pumping of a solution of an active additive is investigated.

One of the promising methods of increasing the oil extraction by flooding oil strata is to use active additives capable of changing the hydrodynamic characteristics of the flow system. The displacement of oil by solutions of active additives was considered earlier in [1-3], and self-similar solutions to problems of frontal displacement were obtained. However, in view of the high cost of the additives, oil is displaced in practice by slugs of solutions of active additives forced by water through the stratum.

The displacement of oil by a slug of an active-additive solution is described by a hyperbolic system of quasilinear equations with discontinuous boundary conditions, which leads to a nonself-similar problem of the interaction of shock waves and rarefaction waves.

1. The process of frontal displacement of oil by a slug of a solution of an active additive can be described in the large-scale approximation by a mixed problem for the system of equations of two-phase flow of two immiscible incompressible fluids and the balance of the additive dissolved in the two phases and adsorbed [1]:

$$\frac{\partial s}{\partial t}+\frac{\partial F}{\partial x}=0, \quad \frac{\partial}{\partial t}\{sc+(1-s)\varphi+a\}+\frac{\partial}{\partial x}\{Fc+(1-F)\varphi\}=0 \tag{1.1}$$

$$s(x,0)=s_*, \quad c(x,0)=c_*, \quad s(0,t)=1, \quad c(0,t)=\begin{cases} c^\circ, & 0<t<1 \\ c_*, & t>1 \end{cases}, \quad F=F(s,c), \quad a=a(c), \quad \varphi=\varphi(c) \tag{1.2}$$

Here, x is a dimensionless coordinate equal to the ratio of the pore volume measured through the stratum from the injection borehole (gallery) to the slug, t is a dimensionless time, the ratio of the volume of the pumped fluid to the volume of the slug, s(x, t) is the saturation of the pore space by the water, c(x, t) is the concentration of the additive in the water solution, φ is the concentration of the additive in the oil, a is the concentration of the adsorbed additive, and F is the Buckley—Leverett function, equal to the fraction of the water in the flow. At the initial time the stratum contains oil and bound water with saturation s_*. Up to the time t = 1 a solution of the additive is pumped through the injection gallery (borehole), and after t = 0 water.

In the case when the active additive improves the displacement conditions (when it is a polymer or a surface-active substance), the final oil extraction increases with increasing c, the curves F(s, c) in Fig. 1 are shifted to the right, and the value of the immovable oil saturation $1 - s^\circ(c)$ decreases [4].

We express the hyperbolic system of quasilinear equations (1.1) in terms of Riemann invariants [5]. By virtue of the relation F = F(s, c), as unknowns for the system we shall consider both (s, c) and (s, F). We shall represent the values of s(x, t) and c(x, t) by points of the plane (s, F) (Fig. 1). To the eigenvalues of the hyperbolic system

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$$\xi_1=F_s, \quad \xi_2=\{F+\varphi'(1-\varphi')^{-1}\}\{s+(\varphi'+a')(1-\varphi')^{-1}\}^{-1}$$

there correspond two families of simple waves

$$\frac{dF}{ds}=F_s=\xi_1, \quad \frac{dF}{ds}=\frac{F+\varphi'(1-\varphi')^{-1}}{s+(\varphi'+a')(1-\varphi')^{-1}}=\xi_2 \tag{1.3}$$

and two families of characteristics

$$\frac{dx}{dt}=\frac{F+\varphi'(1-\varphi')^{-1}}{s+(\varphi'+a')(1-\varphi')^{-1}}, \quad \frac{dc}{dt}=0; \quad \frac{dx}{dt}=F_s, \quad \frac{dI}{dt}=0 \tag{1.4}$$

Here, the second invariant I(s, F) is an arbitrary function constant along the trajectories of the vector field given by the second equation (1.3). Under the hodograph transformation $(x, t) \to \{s(x, t), F(x, t)\}$ the characteristics of the first family (the first equation in (1.4)) go over into simple waves of the first family (the first equation (1.3)). The characteristics of the second family in (1.4) go over into simple waves of the second family in (1.3). A simple wave of the first family on the plane (s, F) corresponds to the line c = const. A simple wave of the second family corresponds to a trajectory whose tangent at the point (s, F) passes through the point $\{-(\varphi'+a')(1-\varphi')^{-1}, -\varphi'(1-\varphi')^{-1}\}$.

The hyperbolic system of conservation laws (1.1) admits discontinuous solutions. The Hugoniot conditions expressing the balance of the mass of the phases and the balance of the additive at a discontinuity for $c^- \neq c^+$ are transformed to [1]

$$V=\{F^{\pm}+[\varphi][c-\varphi]^{-1}\}\{s^{\pm}+[a+\varphi][c-\varphi]^{-1}\}^{-1} \tag{1.5}$$

and for $c^- = c^+$ reduce to the Buckley discontinuity

$$V=(F^+-F^-)(s^+-s^-)^{-1} \tag{1.6}$$

Here, [A] is the difference between the values of A in front of the discontinuity, A^+, and behind it, A^-; V is the velocity of the discontinuity.

The conditions (1.5) mean that the points (s^-, F^-) and (s^+, F^+) lie on a straight line through the point $\{-[\varphi+a][c-\varphi]^{-1}, -[\varphi][c-\varphi]^{-1}\}$ with slope V.

As the conditions of stability of the discontinuity, we take Oleinik's condition [6]: a discontinuity is stable if its velocity is not greater than the velocity of a shock transition from a point behind the discontinuity to any point of the section of the shock adiabat joining the points behind the discontinuity to points in front of it. For the system (1.1), this is the condition for the existence of a discontinuity structure when a capillary discontinuity of interphase pressure is introduced into the system and the relations $a = a(c)$ and $\varphi = \varphi(c)$ of thermodynamic equilibrium are replaced by the equations of the sorption kinetics and the kinetics of solution of the additive in the oil.

For t < 1. the solution to the problem (1.2) is identical to the self-similar solution to the problem of frontal displacement of oil by a solution of the active additive: $s = s(\xi)$, $c = c(\xi)$, $\xi = x/t$ [1, 3].

In the case of a convex sorption isotherm, $a'' \leqslant 0$, and a linear distribution function of the additive between the phases, $\varphi = Kc$, a self-similar solution consists of a centered wave of the first family $c = c^{\circ}$ for $0 \leqslant \xi < V_1$ (in Fig. 1, the points with i = 1, 2, 3, 4 correspond to the values of s_i), a discontinuity from (s_1, c°) to (s_2, c_*) with velocity $\xi = V_1$, a region of rest $s = s_2$, $c = c_*$ for $V_1 < \xi < D = F(s_2, c_*)(s_2 - s_*)^{-1}$, a Buckley shock from s_2 to s_* with velocity $\xi = D$, and a region of rest $s = s_*$, $c = c_*$ for $\xi > D$; here, we have used the notation

$$V_1=F_s(s_1, c^{\circ})=\{F(s_1, c^{\circ})+h\}\{s_1+h+[a][c]^{-1}(1+h)\}^{-1}=\{F(s_2, c_*)+h\}\{s_2+h+[a][c]^{-1}(1+h)\}^{-1}, \quad h=K(1-K)^{-1}$$

On the plane (s, F), this solution corresponds to the path

$$(1, c^{\circ})-J\to(s^{\circ}(c^{\circ}), c^{\circ})-S-(s_1, c^{\circ})-Jc\to(s_2, c_*)-P-J\to(s_*, c_*)$$

where P is the region of rest s = const, c = const; S is a simple wave of the first family, J is the Buckley s discontinuity (1.6), and Jc is the c discontinuity (1.5).

In the case $a'' > 0$, $\varphi = Kc$ a self-similar solution consists of a centered wave of

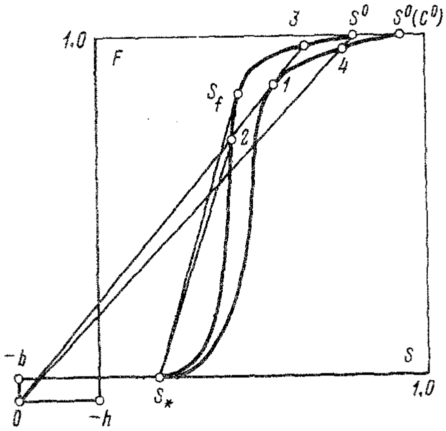

Fig. 1

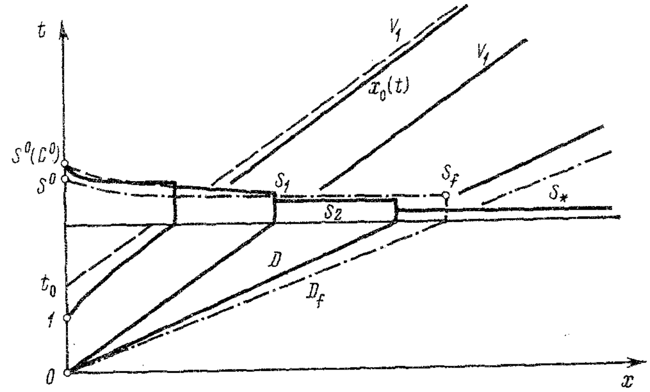

Fig. 2

the first family $c = c^\circ$ for $0 \leqslant \xi < V_1$, a centered wave of the second family from (s_1, c°) to (s_2, c_*) for $V_1 < \xi < V_2$, a region of rest $s = s_2$, $c = c_*$ for $V_2 < \xi < D = F(s_2, c_*)(s_2 - s_*)^{-1}$, a Buckley discontinuity from s_2 to s_* with velocity $\xi = D$, and a region of rest $s = s_*$, $c = c_*$ for $\xi > D$; here, we have used the notation

$$V_1 = F_s(s_1, c^\circ) = \{F(s_1, c^\circ) + h\}\{s_1 + h + a'(c^\circ)(1+h)\}^{-1}, \quad V_2 = \{F(s_2, c_*) + h\}\{s_2 + h + a'(c_*)(1+h)\}^{-1}$$

This solution corresponds to the path

$$(1, c^\circ) - J \to (s^\circ(c^\circ), c^\circ) - S - (s_1, c^\circ) - \dot{C} - (s_2, c_*) - P - J \to (s_*, c_*)$$

on the plane (s, F), where C is a simple wave of the second family.

At the time $t = 1$, the boundary condition $c(0, t)$ is replaced by a discontinuity, which corresponds to termination of injection of the solution and the beginning of the pumping of the water that forces the slug through the stratum. There is a decay of the discontinuity $s^- = 1$, $c^- = c_*$, $s^+ = s^\circ(c^\circ)$, $c^+ = c^\circ$. The decay configuration of this discontinuity contains the discontinuity $(1, c_*) - J \to (s^\circ(c^\circ), c_*)$, so that both when $t > 1$ and $t < 1$ we have $s^+(0, t) = s^\circ(c^\circ)$. This corresponds to the fact that the water is pumped into the stratum after the oil has been washed out by the solution of the active additive in the slug. For $t > 1$, there is interaction of the configuration with the centered wave of the first family of the self-similar solution. The types of the configurations and the solution of the problem (1.2) depend on the form of the functions $a(c)$ and $\varphi(c)$.

2. We consider the case of the Henry sorption isotherm $a = \Gamma c$ and linear distribution function $\varphi = Kc$ of the additive between the phases. Then the second equation (1.1) can be rewritten as

$$\partial\{c(s+b)\}/\partial t + \partial\{c(F+h)\}/\partial x = 0, \quad b = h + \Gamma(1+h) \tag{2.1}$$

Simple waves of the second family go over into contact c discontinuities. The second Riemann invariant has the form $I = (F + h)(s + b)^{-1}$. The condition (1.5) on the discontinuity takes the form $V = I^{\pm}$. The contact discontinuities propagate along the characteristics of the first family.

The decay configuration of the discontinuity of the boundary condition is described by the path $(1, c_*) - J \to (s^\circ(c^\circ), c_*) - P - Jc \to (s^\circ(c^\circ), c^\circ)$. On the contact c discontinuity $x = x_0(t)$ that forms — the back of the slug — there is a complete discontinuity of the concentration $c^+ = c^\circ$, $c^- = c_*$. Since the inequality $x/t = F_s < (F+h)(s+b)^{-1}$ holds in the region of the centered wave, all the characteristics of the second family — the rays of the centered wave — intersect the line of the discontinuity $x = x_0(t)$. They carry the quantity I^+ to the discontinuity line. Hence, we obtain

$$x_0/t = F_s(s^+(x_0), c^\circ), \quad dx_0/dt = I^{\pm}(x_0) \tag{2.2}$$

We integrate Eq. (2.1) over the region of the plane (x, t) bounded by the contour $(0, 0) \to (0, 1) \to (x_0, t) \to (0, 0)$ (Fig. 2). By Green's theorem, it is sufficient for this to integrate the differential form $\theta = c(F+h)dt - c(s+b)dx$ around this contour. The form θ is the flux of the additive:

$$\int_0^1 \theta(0,\eta)\,d\eta+\int_1^t \theta(x_0(\eta),\eta)\,d\eta+\int_t^0 \theta(F_s^+(x_0)\eta,\eta)\,d\eta=0 \qquad (2.3)$$

Substituting in the integral along the line of the discontinuity the expression for the velocity of the discontinuity (the second equation in (2.2)), we find that it is equal to zero. The physical meaning of this fact is that there is no flow of the additive through the line of the contact discontinuity. Therefore, the integral of the form θ along the ray from the point (0, 0) to the point of intersection of the ray with the line of the discontinuity (x_0, t) does not depend on t, i.e., it is a first integral of the motion $x_0(t)$:

$$1+h=\Delta(s^+(x_0),\, c^\circ)t, \quad \Delta(s,\, c)=F+h-(s+b)F_s \qquad (2.4)$$

From the system of the two transcendental equations consisting of the first equation in (2.2) and Eq. (2.4) we can determine $s^+(x_0)$ and x_0 for any t. From the conditions on the discontinuity (the second equation in (2.2)) we find $s^-(x_0)$.

In (2.4), we go to the limit $t \to \infty$. Then $\Delta(s^+(x_0),\, c^\circ) \to 0$, and from the form of the function $F = F(s,\, c^\circ)$ it follows that $s^+(x_0) \to s_1$, $dx_0/dt \to V_1$. On the plane (s, F), the points corresponding to the values of the unknowns in front of the back of the slug and behind it lie on a straight line which passes through the point (—b, —h) on the curves $c = c^\circ$ and $c = c_*$, respectively. As the saturation in front of the back of the slug varies from $s^\circ(c^\circ)$ at t = 1 to s_1 as $t \to \infty$, the saturation behind the back of the slug varies from $s^\circ(c^\circ)$ to s_3 (Fig. 1).

In the region of the driving water $c = c_*$, and the problem (1.2) for the system (1.1) reduces to the mixed problem $s(0,\, t)=s^\circ(c^\circ)$, $s(x_0(t),\, t)=s^-(x_0(t))$ for the first equation in (1.1). In the region $0 < x < x_0$ of the driving water, the values of the saturation $s^-(x_0)$ are carried from the back of the slug along the characteristics in a simple wave of the first family:

$$s(x,\, t')=s^-(x_0(t)), \quad \{x-x_0(t)\}(t'-t)^{-1}=F_s(s^-(x_0(t)),\, c_*) \qquad (2.5)$$

For $s^-(x_0)\geqslant s^\circ=s^\circ(c_*)$ $F^-(x_0)=1$. The saturation $s^-(x_0)$ decreases to the value s° at the time $t^\circ = (1 + h)/\Delta(s_4,\, c^\circ)$, where s_4 is the saturation in front of the discontinuity corresponding to the saturation s° behind it. Therefore, in the region of the driving water for $0 < x < x^\circ = F_s(s_4,\, c^\circ)t^\circ$ the oil is immobile.

We investigate the dynamics of the back of the slug. Since $dx_0/dt < V_1$, the back does not catch up with the front of the slug $x = V_1t$, and the volume $\Omega(t) = V_1t - x_0$ of the slug increases with the time.

We integrate Eq. (2.1) over the region bounded by the contour $(0, 0) \to (0, 1) \to (x_0, t) \to (V_1t, t) \to (0, 0)$, as in the case of (2.3). Since the integrals of the form θ along the lines of the contact discontinuities $x = x_0(t)$ and $x = V_1t$ are equal to zero, we have

$$1+h=\int_{x_0(t)}^{V_1t} \{s(\eta,t)+b\}\,d\eta \qquad (2.6)$$

The obtained expression is the balance of the additive in the slug. Going to the limit $t \to \infty$ in the expression, we obtain $\Omega(\infty)=(1+h)(s_1+b)^{-1}$. With the passage of time, the volume of the slug is stabilized. From the time t = 1, the volume of the slug increases by $t_0 = (1 + h)(F_1 + h)^{-1}$ times. The back of the slug has the inclined asymptote $x = V_1(t - t_0)$.

3. The region of the flow can be divided into five zones (Fig. 2):

1°. $x > Dt$, the zone of the displaced oil, $s = s_*$, $c = c_*$.

2°. $V_1t < x < Dt$, the water—oil bank in front of the slug, $s = s_2$, $c = c_*$.

3°. $x_0 < x < V_1t$, the slug. The profile of the saturation distribution corresponds to a path on the plane (s, F), namely, the section of the curve $c = c^\circ$ from s_1 to $s^+(x_0)$.

4°. $x^\circ < x < x_0$, the zone of the driving water with mobile oil phase. The path is the section of the curve $c = c_*$ from $s^-(x_0)$ to s°.

5°. $0 < x < x^\circ$, the zone of the driving water with immobile oil phase. The path is the section of the curve $c = c_*$ from s° to $s^\circ(c^\circ)$.

With the passage of time, the first, second, and fourth zones increase unboundedly in size, while the third zone stabilizes as $t \to \infty$ and the fifth as $t = t^\circ$. In the limit $t \to \infty$, the saturation in the slug decreases to s_1, and in the zone of the driving water it varies from $s^\circ(c^\circ)$ at the injection borehole to s_3 at the back of the slug.

Figure 2 shows the profiles of the saturation distribution in the case of displacement by a slug (continuous curve), an active additive (broken curve), and water (chain curve). Compared with ordinary flooding, the water saturation of the production in the case of displacement by a slug is lower in the stage of the water—oil bank and the slug, the flooding begins later, and the final oil extraction is higher. In the case of displacement by a slug, the saturation in a certain region behind the back of the slug is higher than in the case of displacement by a solution of the additive, and in the central part of the zone of the driving water it is lower. At large volumes of the slug, complete flooding commences at the time when the back of the slug reaches the extraction gallery, and the process of extract ends. For smaller volumes, the process of extraction can be continued after this time. If a given level of water saturation is achieved before the arrival of the back of the slug, then the oil extraction is the same as in the case of displacement by a solution of the additive. Otherwise, the oil extraction is lower.

In the case of a high concentration of the additive solution, $s_3 > s^\circ$ [2], the fourth zone is absent, and all the oil in the zone of the driving water is immobile.

In the case $F_c > 0$ (the additive results in less favorable displacement conditions) or $c_* > c^\circ$ (the slig demineralizes the stratal water), the behavior of the back of the slug is described by the system (2.2), (2.4) until the emergence from the centered wave, after which its velocity becomes equal to V_1, and the volume of the slug is stabilized.

4. We consider the process of cyclic injection into a stratum of a solution of an active additive [7]:

$$c(0,t)=\begin{cases} c^\circ, & 0<t<1, \quad t_1<t<t_3,\ldots \\ c_*, & 1<t<t_1, \quad t_3<t<t_5,\ldots \end{cases}$$

At $t = t_1$, there is decay of the discontinuity of the boundary condition with the configuration $(1, c^\circ) - J \to (s^\circ(c^\circ), c^\circ) - P - Jc \to (s^\circ(c^\circ), c_*)$ and interaction with a simple wave of the first family (2.5) begins. The resulting c discontinuity — the front of a second slug — propagates along a characteristic of the first family. Along it, there is a complete discontinuity of the concentration $c^-(x_1) = c^\circ$, $c^+(x_1) = c_*$. In the region of the simple wave $\{x - x_0(t)\}(t' - t)^{-1} = F_s(s^-(x_0), c_*) < I(x_0)$, and therefore the front of the second slug intersects all the characteristics of the second family. They carry to the line $x = x_1(t')$ the quantity I^-. At the same time,

$$\{x_1(t')-x_0(t)\}(t'-t)^{-1}=F_s(s^-(x_0(t)),\ c_*) \tag{4.1}$$

We integrate Eq. (2.1) over the region of the plane (x, t) bounded by the contour $(0, 0) \to (0, t_1) \to (x_1(t'), t') \to (x_0(t), t) \to (0, 0)$, as in the case of (2.3). The integral of the form θ along the line of the contact discontinuity $x = x_1(t')$ is equal to zero. Therefore, the integral of the form θ along the broken line $(0, 0) - (x_0(t), t) - (x_1(t'), t')$ does not depend on the time t' and is a first integral of the motion $x = x_1(t')$:

$$(1+h)t_1=\Delta(s^+(x_0),\ c^\circ)t+\Delta(s^-(x_0),\ c_*)(t'-t) \tag{4.2}$$

From the system of the five transcendental equations (2.2), (2.4), (4.1), and (4.2) for any t' we can determine t, x_0, $s^+(x_0)$, $s^-(x_0) = s^+(x_1)$, and x_1. From the conditions on the discontinuity, we find $s^-(x_1)$. Since $c^-(x_1) = c^+(x_0)$, it follows that $s^-(x_1) = s^+(x_0)$.

In the limit $t' \to \infty$, we have $s^+(x_1) \to s_3$, $s^-(x_1) \to s_1$, $dx_1/dt' \to V_1$. In the region of the second slug $c = c^\circ$, and the values of the saturation are carried from the front of the slug $s^-(x_1)$ along the characteristics in a simple wave of the first family as in the case of (2.5).

On the back of the second slug $x = x_2(t'')$ there is a complete discontinuity of the concentration, $c^- = c_*$, $c^+ = c^\circ$. Integration of the form θ along the contour

$$(0,\ 0)\to(0,\ t_3)\to(x_2(t''),\ t'')\to(x_1(t'),\ t')\to(x_0(t),\ t)\to(0,\ 0)$$

leads to an expression analogous to (2.4) and (4.2). The dynamics of the back of the slug is described by a system of seven transcendental equations. As $t'' \to \infty$, we have $s^+(x_2) \to s_1$, $s^-(x_2) \to s_3$, $dx_2/dt'' \to V_1$. In the zone of the driving water $c = c_*$, and the values of the saturation are carried from the back of the slug $s^-(x_2) = s^-(x_0)$ along the characteristics of the second family as in the case of (2.5).

The characteristics in the zones $c = c^\circ$ and $c = c_*$ corresponding to the saturations $s^+(x_0) = s^-(x_1)$ and $s^-(x_0) = s^-(x_2)$ are pairwise parallel. The points $x_0(t)$, $x_1(t)$, and $x_2(t)$ lie on the characteristics of the second family, along which $s^\pm(x_2(t)) > s^\pm(x_1(t)) > s^\pm(x_0, (t))$, and therefore $dx_2/dt < dx_1/dt < dx_0/dt$ and the volumes of the zone between the slugs and of the second slug increase. From the condition of mass balance we find, as in the case of (2.6), that in the limit $t \to \infty$

$$x_1-x_0 \to (1+h)(t_1-1)(s_3+b)^{-1}, \quad x_2-x_1 \to (1+h)(t_2-t_1)(s_1+b)^{-1}$$

The discontinuities x_1 and x_2 have inclined asymptotes:

$$x=V_1(t-t_0-t_2), \quad t_2=(1+h)(t_1-1)(F_3+h)^{-1}, \quad x=V_1(t-t_0-t_2-t_4), \quad t_4=(1+h)(t_3-t_1)(F_1+h)^{-1}$$

The problem can be solved similarly for arbitrary piecewise constant $c(0, t)$. In the contact case $a = \Gamma c$ and $\varphi = Kc$, each successive discontinuity propagates in the field of the characteristics of the second family behind the preceding discontinuity. The motion of each slug is independent of the concentrations of the solutions pumped after it.

For initial water saturation $s_* < s(x, 0) < s_3$ of the stratum, the solution to the problem of cyclic injection differs from the case $s(x, 0) = s_*$ only by the self-similar part of the solution ahead of the front $x = V_1(t)$. Depending on the value of $s(x, 0)$, there are four types of solution differing in the sequence of the motion along $c = c_*$ and the J discontinuity from the point (s_2, c_*). For $s_3 < s(x, 0) < s^\circ$, the solution of the displacement problem is identical to the solution for $s(x, 0) = s_*$ in front of the broken line consisting of the characteristics of the second family corresponding to $s(x, 0) = s^-(x_0) = s^+(x_1) = \ldots$. The volumes of the slugs and the velocities of the fronts stabilize after a finite time.

5. We now consider displacement of oil by a slug when there is an irreversible retention of some of the active additive by the skeleton of the porous medium. We assume that the desorption isotherm is a linear function of the concentration,

$$a^\circ(c, c^\circ)=\{\Gamma c^\circ-a^\circ(c_*, c^\circ)\}(c^\circ-c_*)^{-1}(c-c_*)+a^\circ(c_*, c^\circ),$$

where c° is the concentration of the solution at the beginning of the desorption process. For $\partial c/\partial t < 0$ and $c^- < c^+$, the constant b in Eq. (2.1) and in the mass balance conditions on the discontinuity is $b^\circ=\{K+(\Gamma c^\circ-a^\circ(c_*, c^\circ))(c^\circ-c_*)^{-1}\}(1+h)$, and the point 0 is shifted to the right (Fig. 3).

At $t = 1$, the interaction of the discontinuity of the boundary condition $c^- = c_*$, $c^+ = c^\circ$ with the centered wave of the self-similar solution begins. In the region of the centered wave, we have

$$x_0/t=F_s(s^+(x_0), c^\circ)<\{F^+(x_0)+h\}\{s^+(x_0)+b\}^{-1}=dx_0/dt$$

and therefore the back of the slug intersects all rays of the centered wave. Integrating Eq. (2.1) over the region bounded by the contour $(0, 0) \to (0, 1) \to (x_0, t) \to (0, 0)$ using the conditions on the discontinuity, we obtain

$$1+h=\Delta^\circ(s^+(x_0), c^\circ)t, \quad \Delta^\circ(s, c)=F+h-(s+b^\circ)F_s \tag{5.1}$$

In contrast to the case of reversible sorption, at the front of the slug

$$dx_0/dt=(F_1+h)(s_1+b^\circ)^{-1}>(F_1+h)(s_1+b)^{-1}=V_1$$

Therefore, at a certain time t_1 the back of the slug catches up with its front. From (5.1), we obtain an expression for t_1:

$$t_1=(1+h)/\Delta^\circ(s_1, c^\circ)=(1+h)/V_1(b-b^\circ)$$

which expresses equality of the mass of the pumped additive and the additive retained by the porous medium in the volume V_1t_1. At the time t_1, the slug disappears (Fig. 4).

Behind the back of the slug $c = c_*$, the flow takes place in the pore space with

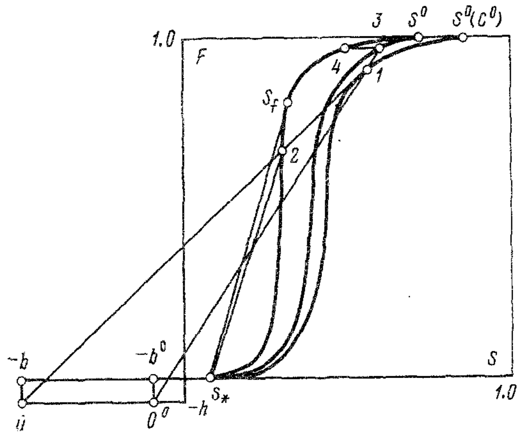

Fig. 3

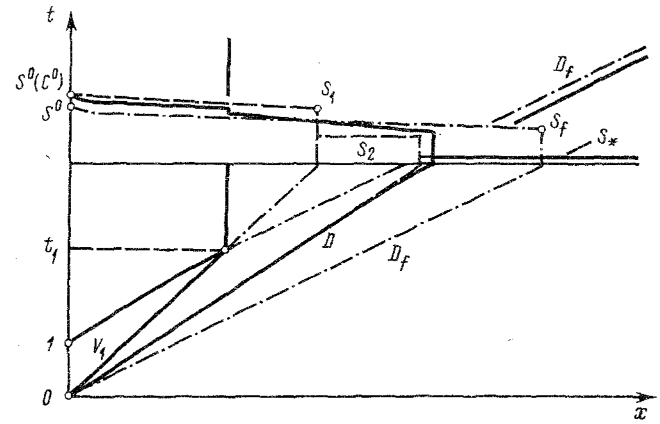

Fig. 4

irreversibly adsorbed additive. The water permeability remains lowered, and for the oil it becomes the same as in front of the slug [4]. Therefore, the flow in the region $x < x_0$ traversed by the slug is described by the first equation in (1.1) with Buckley–Leverett function $F^a(s, c_*)$ for which $F(s, c^\circ) < F^a(s, c_*) < F(s, c_*)$. In the region ahead of the boundary of the porous medium with retained additive and without it, $x = V_1 t_1$, the flow is described by the first equation in (1.1) with the Buckley–Leverett function $F(s, c_*)$.

Along the back of the slug there is a discontinuity from the curve $F = F^a(s, c_*)$ to the curve $F = F(s, c^\circ)$. The points behind the back of the slug and in front of it lie on a straight line which passes through the point $0^\circ(-b^\circ, -h)$. From the condition on the discontinuity we find $s^-(x_0)$.

In the region traversed by the slug, the values of the saturation $s^-(x_0)$ propagate along characteristics as in the case of (2.5) to the line $x = V_1 t_1$:

$$s(V_1t_1, t') = s^-(x_0(t)), \quad \{V_1t_1 - x_0(t)\}(t'-t)^{-1} = F_s{}^a(s^-(x_0), c_*)$$

On the line $x = V_1 t_1$, there is a discontinuity of the saturation, at which the balance $F^a(s^-(V_1t_1, t'), c_*) = F(s^+(V_1t_1, t'), c_*)$ of the masses of the phases holds. On the plane (s, F), this corresponds to a jump from the curve $F = F^a(s, c_*)$ to the curve $F = F(s, c_*)$ parallel to the abscissa. As $s^-(V_1t_1, t')$ changes from s_3 at $t = t_1$ to s° as $t \to \infty$, the value of $s^+(V_1t_1, t')$ varies from s_4 to s° (Fig. 3).

At the point (V_1t_1, t_1), the discontinuity $s^- = s_4$, $s^+ = s_2$, $c^\pm = c_*$ decays, this being realized by the centered wave $(x - V_1t_1)(t - t_1)^{-1} = F_s(s, c_*)$. In the region ahead of the line $x = V_1 t_1$ to the trailing edge of the centered wave the values of $s^+(V_1t_1, t')$ propagate along the characteristics as in the case of (2.5).

At the time t_2, the front of the water–oil bank $x = Dt$ catches up with the leading edge of the centered wave, $Dt_2 = F_s(s_2, c_*)(t_2 - t_1) + V_1t_1$. The discontinuity begins to interact with the centered wave

$$\{x_1(t) - V_1t_1\}(t - t_1)^{-1} = F_s(s^-(x_1), c_*) \tag{5.2}$$

Since

$$dx_1/dt = F(s^-(x_1), c_*)\{s^-(x_1) - s_*\}^{-1} < F_s(s^-(x_1), c_*)$$

the characteristics carry values $s^-(x_1) > s_f$ from the centered wave to the line of the discontinuity. We integrate the first equation in (1.1) over the region of the plane (x, t) bounded by the contour $(V_1t_1, t_1) \to (x_1(t), t) \to (Dt_2, t_2) \to (V_1t_1, t_1)$. The corresponding differential form is $\Theta_s = F dt - s dx$. We obtain a first integral of the motion $x = x_1(t)$ in the form

$$\Delta_s(s^-(x_1), c_*)(t - t_1) = \Delta_s(s_2, c_*)(t_2 - t_1), \quad \Delta_s(s, c) = F - (s - s_*)F_s. \tag{5.3}$$

From the system of the two transcendental equations (5.2) and (5.3) we obtain x_1 and $s^-(x_1)$ for any t. In the limit $t \to \infty$, $s^-(x_1) \to s_f$ and $dx_1/dt \to D_f$.

Since $dx_1/dt < D_f$, the leading edge of the centered wave does not catch up with the front of the water–oil bank. The volume of the region between them increases and stabilizes as $t \to \infty$. Integrating the form Θ_s around the contour

$$(V_1t_1, t_1) \to (V_1t_1+D_f(t-t_1), t) \to (x_1(t), t) \to (Dt_2, t_2) \to (V_1t_1, t_1)$$

and going to the limit $t \to \infty$, we obtain as in the case of (2.6) the inclined asymptote of the front of the water—oil bank:

$$x=V_1t_1+D_f(t-t_1)+\Delta_s(s_2, c_*)(t_2-t_1)(s_f-s_*)^{-1}$$

Figure 4 shows the profiles of the saturation in the case of displacement by a slug (continuous line) by a solution of additive (broken line), and by water (chain line). For small volumes of the slug (the axis of the recovery gallery lies to the right of $x = V_1t_1$), the slug disappears without reaching the recovery gallery. Ahead of the ray $x = V_1t$ the saturation in the case of displacement by a slug is higher than in the case of flooding, and behind the ray $x = V_1t$ it is lower, i.e., the use of a slug leads to a slowing down in the growth of the water saturation of the production in the initial stage of the flooding. In the case of displacement by a slug, the saturation varies through the stratum from s_* to $s^\circ(c^\circ)$, and in the case of flooding from s_* to s°, i.e., the use of a slug increases the final oil recovery. At the same time, the period of waterless exploitation increases somewhat.

At large volumes of the slug (the axis of the recovery gallery lies to the left of $x = V_1t_1$) the exploitation indices in the case of displacement by a slug are the same as in the case of displacement by a solution of additive.

In the case of strong sorption or an initial saturation $s(x, 0)$ such that $s_f < s_2$ the velocity of the front of the water—oil bank is equal to D_f and the discontinuity $x = D_ft$ does not interact with the centered wave. The use of a slug does not change the time at which flooding begins.

6. We now consider briefly the case of a nonlinear sorption isotherm. For the Langmuir isotherm $a'' < 0$ and $\varphi = Kc$ the decay configuration of the discontinuity of the boundary condition at the time $t = 1$ has the form $(1, c_*)-J\to(s^\circ(c^\circ), c_*)\bar{-}P\bar{-}C-(s^\circ(c^\circ), c^\circ)$. Interaction of the centered wave of the second family which is formed with the centered wave of the first family begins. There arises a zone with decreasing concentration which adjoins the wave of the first family along the c° characteristic $x_0(t)$ and the zone $c = c_*$ of the driving water along the c_* characteristic $x_2(t)$ of the back of the slug. The motion of x_0 is described by the system of the two equations (2.2) and (2.4) for $b = h + a'(c^\circ)(1 + h)$. At the time $t_1 = (1 + h)/\Delta(s_1, c^\circ)$, the front of the slug catches up with the line x_0. The region of the slug with maximal concentration disappears, and the washing out of the slug by the driving water commences.

Behind the front of the slug $\xi_1 < \xi_2$, and the characteristics of the second family intersect all characteristics of the first. To investigate the behavior of the characteristics, we construct a hodograph transformation. The velocity of the characteristics of the first family increases but at the second decreases.

At $t = t_1$, the perturbation imposed on the configuration of the self-similar solution by the discontinuity of the boundary condition catches up with the discontinuity $x = V_1t$, and interaction with the front of the slug commences. On the front of the slug, the line $x = x_1(t)$, the Jouguet condition [8]

$$dx_1/dt=(F^{\pm}+h)\{s^{\pm}+h+[a][c]^{-1}(1+h)\}^{-1}=F_s^{-}, \quad c^{+}=c_*=0$$

is satisfied as well as the mass balance conditions.

Under the hodograph transformation, the line $x_1 + 0$ goes over into the section of the curve $c = c_*$ from s_2 at $t = t_1$ to s_4 at $t \to \infty$, where

$$F_s(s_4, c_*)=(F_4+h)\{s_4+h+a'(c_*)(1+h)\}^{-1}=V_4.$$

The line $x_1 - 0$ goes over into the section of the Jouguet line $F_s^{-}=(F^{\pm}+h)\{s^{\pm}+h+[a][c]^{-1}(1+h)\}^{-1}$ from s_1 at $t = t_1$ to s_4 at $t \to \infty$. The line x_2 goes over into the section of the curve $c = c_*$ from $s^\circ(c^\circ)$ at $t = 1$ to s_4 as $t \to \infty$. The saturation in the slug decreases to s_4. The concentration of the additive increases from 0 at the back of the slug x_2 to $c^-(x_1)$ at the front. Integrating the second equation in (1.1) over the region bounded by the contour $(0, 1) \to (x_1, t) \to (V_1t_1, t_1) \to (0, 1)$, we obtain $x_1(t)=\{a(c^-)-a'(c^-)c^-\}^{-1}$ for $c^- = c^-(x_1)$. In the limit $t \to \infty$, we therefore have $c^-(x_1)\sim\{-a''(0)V_4t/2\}^{-1/2}$. The volume of the slug increases unboundedly.

In the case of weak sorption ($s_2 < s_f$), the simple wave of the first family ahead of the front of the slug interacts with the front of the water—oil bank, changing its velocity and the saturation from D, s_2 to D_f, s_f as $t \to \infty$. When $s_2 > s_f$, this interaction does not occur.

A similar solution to the problem (1.2) holds in the case $a=\Gamma c$, $\varphi''<0$.

For a concave sorption isotherm, $a'' > 0$, and $\varphi = Kc$ at the time $t = 1$ a discontinuity $x_0(t)$ forms, along which $c^- = c_* = 0$, $c^+ = c^\circ$. The motion of the discontinuity is decribed by the system of equations (2.2) and (2.4) for $b=h+[a][c]^{-1}(1+h)$. At the time $t_1 = (1 + h)/\Delta(s_1, c^\circ)$, the discontinuity catches up with the boundary $x = V_1 t$ of the first and second waves of the self-similar solution. Under the hodograph transformation, the line $x_0 + 0$ goes over into the section of the simple wave of the second family from s_1 at $t = t_1$ to s_2 as $t \to \infty$. The line $x_0 - 0$ goes over into the section of the curve $c = c_*$ from $s^\circ(c^\circ)$ at $t = 1$ to s_2 as $t \to \infty$. Integrating the second equation (1.1) over the region bounded by the contour $(0, 0) \to (0, 1) \to (x_0, t) \to (0, 0)$ for $t > t_1$ we obtain for $c^+ = c^+(x_0)$ the expression $x_0(t)=\{a'(c^+)c^+-a(c^+)\}^{-1}$. As $t \to \infty$, the saturation in the slug decreases to s_2, $dx_0/dt \to V_2$, and the concentration of the additive increases from 0 on the front to $c^+(x_0)\sim\{a''(0)V_2t/2\}^{-1/2}$ at the back of the slug. The volume of the slug increases unboundedly: $\Omega(t)\sim(1+h)(F_2+h)^{-1}(2a''(0)V_2^3t)^{1/2}$.

The problem (1.2) has a similar solution for $a = \Gamma c$ and $\varphi'' > 0$.

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