CHAPTER V

SOME SIMPLE EXAMPLES OF INFERENCES INVOLVING PROBABILITY AND LIKELIHOOD

1. The logical consequences of uncertainty

The concepts sketched in Chapter III have arisen in the study of numerical observations in the Natural Sciences; they are intended for use in the inferences by which progress in the sciences is guided. Since the reasoning is quantitative it involves mathematical operations, which need not, however, be of a very complicated kind. Indeed, the examples may be confined to simple cases, though often at the expense of being scientifically trivial, for it is not the mathematics but the logical nature of these concepts that requires to be exemplified. Since the reasoning is inductive, the development for it of appropriate mathematical operations seems to run counter to the view that all mathematics can be reduced to a single and wholly deductive system. Admittedly deductive processes play a predominant part in mathematics, yet it is difficult to admit that mathematics is less than the whole art of exact quantitative reasoning, and as such must extend beyond the domain of deduction proper.

The theory that all mathematics could be reduced to a purely deductive system, which was popular about the beginning of this century, has, moreover, in the meantime suffered, with the development of axiomatic studies, some rather severe setbacks. It is

common ground that the consistency of the axiomatic basis of a deductive system is essential for the reliability of its consequences. It has been formally demonstrated that a system admitting one contradiction must admit all, in the sense that any proposition whatever can be deduced from it, by formally rigorous processes. The non-existence of contradictory consequences is thus a burning question for the whole superstructure. Moreover, it has been proved that the non-existence of such contradictions can never be demonstrated on the basis of the axioms of the system themselves. It would be rather absurd, indeed, to imagine that any chain of theorems, derived from a given axiomatic basis, could disprove a possible property of that basis, when it is known that, if it had that property, these same theorems could certainly be deduced from it. For the possibility of proving such theorems does not depend upon the truth of what they assert. It would seem, therefore, that the validity of a purely deductive system has at best the same logical status as has a scientific theory, which has not yet been found in any case to be in conflict with the observations. As such it appears to be solidly based on a well-tested induction.

The axiomatic theory of mathematics has not been taken very seriously in those branches of the subject in which applications to real situations are in view. For, in applied mathematics, it is unavoidable that new concepts should from time to time be introduced as the cognate science develops, and any new definition having axiomatic implications is inevitably a threat to the internal consistency of the whole system of axioms into which it is to be incorporated.