Comparing Trig and Hyperbolic Trig Functions

By the Maths Learning Centre, University of Adelaide

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<tr><th>Trigonometric Functions</th><th>Hyperbolic Trigonometric Functions</th></tr>
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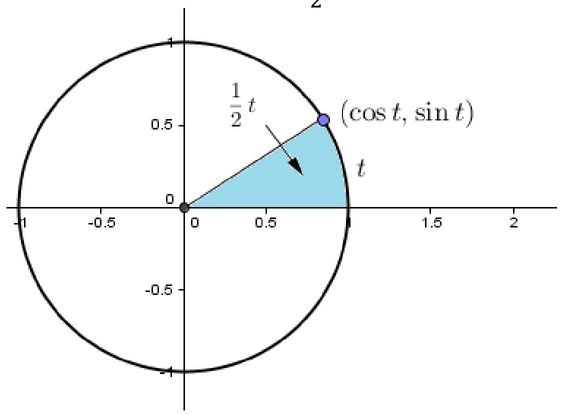
Definition using unit circle:
If a point is an arc length of t anticlockwise around the unit circle from (1,0), then that point is $(\cos t, \sin t)$.
(Note the line segment from the origin to the unit circle sweeps out an area of $\frac{1}{2}t$.)



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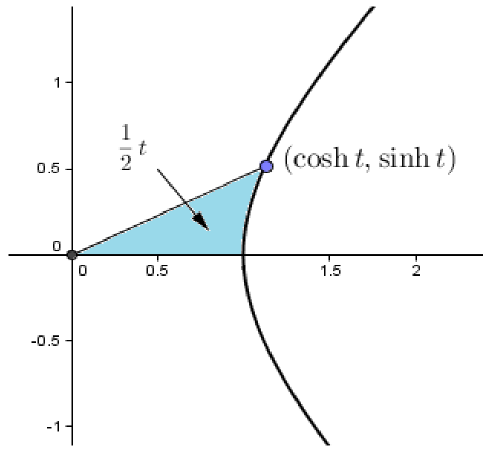
Definition using unit hyperbola:
If a line segment from the origin to the unit hyperbola sweeps out an area of $\frac{1}{2}t$ between (1,0) and a particular point as it moves upwards, then that point is $(\cosh t, \sinh t)$.



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Parameterising a curve:
If we use all values of t, the points $(\cos t, \sin t)$ form the circle with equation $x^2 + y^2 = 1$.

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Parameterising a curve:
If we use all values of t, the points $(\cosh t, \sinh t)$ form the right-hand branch of the hyperbola with equation $x^2 - y^2 = 1$.

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Defining other functions:

$$\tan x = \frac{\sin x}{\cos x} \qquad \operatorname{cosec} x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

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Defining other functions:

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \operatorname{cosech} x = \frac{1}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}$$

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Basic identities

$$(\cos x)^2 + (\sin x)^2 = 1$$
$$1 + (\tan x)^2 = (\sec x)^2$$
$$(\cot x)^2 + 1 = (\operatorname{cosec} x)^2$$

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Basic identities

$$(\cosh x)^2 - (\sinh x)^2 = 1$$
$$1 - (\tanh x)^2 = (\operatorname{sech} x)^2$$
$$(\coth x)^2 - 1 = (\operatorname{cosech} x)^2$$

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Double angle identities

$$\sin(2x) = 2\sin x \cos x$$
$$\cos(2x) = (\cos x)^2 - (\sin x)^2$$
$$= 2(\cos x)^2 - 1$$
$$= 1 - 2(\sin x)^2$$

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Double area identities

$$\sinh(2x) = 2\sinh x \cosh x$$
$$\cosh(2x) = (\cosh x)^2 + (\sinh x)^2$$
$$= 2(\cosh x)^2 - 1$$
$$= 1 + 2(\sinh x)^2$$

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Trigonometric Functions	Hyperbolic Trigonometric Functions
Odd and even properties $\cos x$ is an even function $\sin x$ is an odd function $\tan x$ is an odd function	**Odd and even properties** $\cosh x$ is an even function $\sinh x$ is an odd function $\tanh x$ is an odd function
Values at zero $\cos(0) = 1 \quad \sin(0) = 0 \quad \tan(0) = 0$	**Values at zero** $\cosh(0) = 1 \quad \sinh(0) = 0 \quad \tanh(0) = 0$
Derivatives $\frac{d}{dx}\cos x = -\sin x$ $\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\tan x = (\sec x)^2$	**Derivatives** $\frac{d}{dx}\cosh x = \sinh x$ $\frac{d}{dx}\sinh x = \cosh x$ $\frac{d}{dx}\tanh x = (\operatorname{sech} x)^2$
Derivatives of inverse functions $\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$	**Derivatives of inverse functions** $\frac{d}{dx}\operatorname{acosh} x = \frac{1}{\sqrt{x^2-1}}$ $\frac{d}{dx}\operatorname{asinh} x = \frac{1}{\sqrt{1+x^2}}$ $\frac{d}{dx}\operatorname{atanh} x = \frac{1}{1-x^2}$
Maclaurin Series $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$	**Maclaurin Series** $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$ $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$
Relationship to e^x $e^{ix} = \cos x + i\sin x$ $\cos x = \frac{e^{ix} + e^{-ix}}{2}$ $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$	**Relationship to e^x** $e^x = \cosh x + \sinh x$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\sinh x = \frac{e^x - e^{-x}}{2}$