USEFUL TRIGONOMETRIC IDENTITIES

Definitions

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sec x = \frac{1}{\cos x} \qquad \operatorname{cosec} x = \frac{1}{\sin x} \qquad \cot x = \frac{1}{\tan x}$$

Fundamental trig identity

$$(\cos x)^2 + (\sin x)^2 = 1$$

$$1 + (\tan x)^2 = (\sec x)^2$$

You can get this one from the top one if you divide by $(\cos x)^2$

$$(\cot x)^2 + 1 = (\operatorname{cosec} x)^2$$

You can get this one from the top one if you divide by $(\sin x)^2$

Odd and even properties

$$\cos(-x) = \cos(x) \qquad \sin(-x) = -\sin(x) \qquad \tan(-x) = -\tan(x)$$

cos is an even function

sin and tan are odd functions

Double angle formulas

$$\sin(2x) = 2 \sin x \cos x \qquad \cos(2x) = (\cos x)^2 - (\sin x)^2$$

$$\cos(2x) = 2(\cos x)^2 - 1$$

You get this from the top one if you sub in $(\sin x)^2 = 1 - (\cos x)^2$

$$\cos(2x) = 1 - 2(\sin x)^2$$

You get this from the top one if you sub in $(\cos x)^2 = 1 - (\sin x)^2$

Half angle formulas

$$\left[\sin(\tfrac{1}{2}x)\right]^2 = \tfrac{1}{2}(1 - \cos x)$$

$$\left[\cos(\tfrac{1}{2}x)\right]^2 = \tfrac{1}{2}(1 + \cos x)$$

These come from rearranging the cos double angle formulas and then replacing x with $\frac{1}{2}x$

Sums and differences of angles

If you sub $A = x$, $B = x$ into these you get the double angle formulas

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

You can get these by replacing B with $-B$ in the other two and using the odd and even properties

** See other side for more identities **

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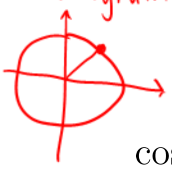
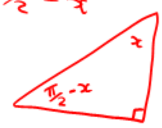
You can find these by drawing a diagram of the unit circle



Unit circle properties

$$
\begin{aligned}
\cos(\pi - x) &= -\cos(x) & \sin(\pi - x) &= \sin(x) & \tan(\pi - x) &= -\tan(x) \\
\cos(\pi + x) &= -\cos(x) & \sin(\pi + x) &= -\sin(x) & \tan(\pi + x) &= \tan(x) \\
\cos(2\pi - x) &= \cos(x) & \sin(2\pi - x) &= -\sin(x) & \tan(2\pi - x) &= -\tan(x) \\
\cos(2\pi + x) &= \cos(x) & \sin(2\pi + x) &= \sin(x) & \tan(2\pi + x) &= \tan(x)
\end{aligned}
$$

You can find these by drawing a right-angled triangle with small angles x and $\pi/2 - x$



Right-angled triangle properties

$$
\cos\left(\tfrac{\pi}{2} - x\right) = \sin(x) \qquad \sin\left(\tfrac{\pi}{2} - x\right) = \cos(x)
$$

These are a combination of the above two sets of formulas and the odd/even properties

Shifting by $\frac{\pi}{2}$

$$
\begin{aligned}
\cos(x) &= \cos(x) & \cos(x) &= \cos(x) & \cos(-x) &= \cos(x) \\
\cos(x + \tfrac{\pi}{2}) &= -\sin(x) & \cos(x - \tfrac{\pi}{2}) &= \sin(x) & \cos(\tfrac{\pi}{2} - x) &= \sin(x) \\
\cos(x + \pi) &= -\cos(x) & \cos(x - \pi) &= -\cos(x) & \cos(\pi - x) &= -\cos(x) \\
\cos(x + \tfrac{3\pi}{2}) &= \sin(x) & \cos(x - \tfrac{3\pi}{2}) &= -\sin(x) & \cos(\tfrac{3\pi}{2} - x) &= -\sin(x) \\
\cos(x + 2\pi) &= \cos(x) & \cos(x - 2\pi) &= \cos(x) & \cos(2\pi - x) &= \cos(x)
\end{aligned}
$$

$$
\begin{aligned}
\sin(x) &= \sin(x) & \sin(x) &= \sin(x) & \sin(-x) &= -\sin(x) \\
\sin(x + \tfrac{\pi}{2}) &= \cos(x) & \sin(x - \tfrac{\pi}{2}) &= -\cos(x) & \sin(\tfrac{\pi}{2} - x) &= \cos(x) \\
\sin(x + \pi) &= -\sin(x) & \sin(x - \pi) &= -\sin(x) & \sin(\pi - x) &= \sin(x) \\
\sin(x + \tfrac{3\pi}{2}) &= -\cos(x) & \sin(x - \tfrac{3\pi}{2}) &= \cos(x) & \sin(\tfrac{3\pi}{2} - x) &= -\cos(x) \\
\sin(x + 2\pi) &= \sin(x) & \sin(x - 2\pi) &= \sin(x) & \sin(2\pi - x) &= -\sin(x)
\end{aligned}
$$

$$
\begin{aligned}
\tan(x) &= \tan(x) & \tan(x) &= \tan(x) & \tan(-x) &= -\tan(x) \\
\tan(x + \tfrac{\pi}{2}) &= -\cot(x) & \tan(x - \tfrac{\pi}{2}) &= -\cot(x) & \tan(\tfrac{\pi}{2} - x) &= \cot(x) \\
\tan(x + \pi) &= \tan(x) & \tan(x - \pi) &= \tan(x) & \tan(\pi - x) &= -\tan(x) \\
\tan(x + \tfrac{3\pi}{2}) &= -\cot(x) & \tan(x - \tfrac{3\pi}{2}) &= -\cot(x) & \tan(\tfrac{3\pi}{2} - x) &= \cot(x) \\
\tan(x + 2\pi) &= \tan(x) & \tan(x - 2\pi) &= \tan(x) & \tan(2\pi - x) &= -\tan(x)
\end{aligned}
$$

**** See other side for more identities ****