

4-Dimensional Noughts and Crosses

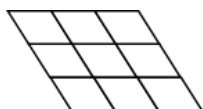
By Dr David Butler

Maths Learning Centre, The University of Adelaide

Introduction

4D Noughts and Crosses is an extension of standard noughts and crosses, but played on a 4-dimensional board. It is designed to look cool when you play it on a whiteboard, and to be fuel for discussion about multi-dimensional space. Playing it will help you “see” objects in multidimensional space, and to realise that the “fourth dimension” doesn’t have to be time. Plus of course it’s fun too.

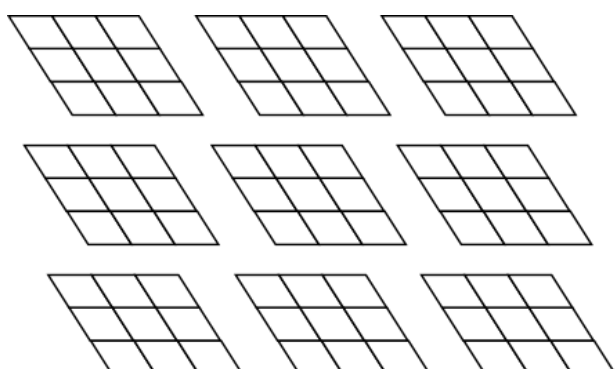
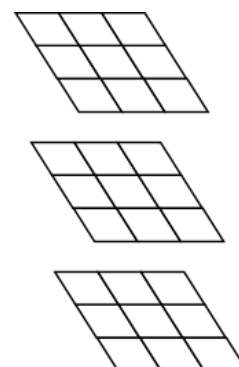
About the 4D noughts and crosses board



This is an ordinary 2-dimensional noughts and crosses board. To play the game two players take turns placing their symbol (either an O or an X) in a space and attempt to make three symbols in a line, either horizontally, vertically or diagonally.

You can make a 3-dimensional noughts and crosses board by stacking three 2D noughts and crosses boards on top of each other. Let’s call each of these 2D boards a “flat”.

You imagine the matching positions in each flat as being directly above each other. Now as well as the lines in each 2D board there are many lines that run vertically and diagonally downwards through the three flats. Also, there are not just lines but *planes*: collections of 9 spaces that line up to make a copy of a flat – some horizontal, some vertical, and some diagonal.



Finally, this is a 4-dimensional noughts and crosses board – you make it by taking three 3D noughts and crosses boards and placing them next to each other. Let’s call each of these 3D boards a “stack”. You imagine the matching positions in each stack as being lined up across a fourth dimension.

Now as well as the lines and planes in each separate stack there are many new lines that run across the three stacks, and many new planes that run across the three stacks (a list of all of the planes is included later).

How the game works

Players: 4D Noughts and Crosses is a game for two players.

Setting up: Draw the 4D Noughts and Crosses board, and choose a different symbol (O or X) for each player.

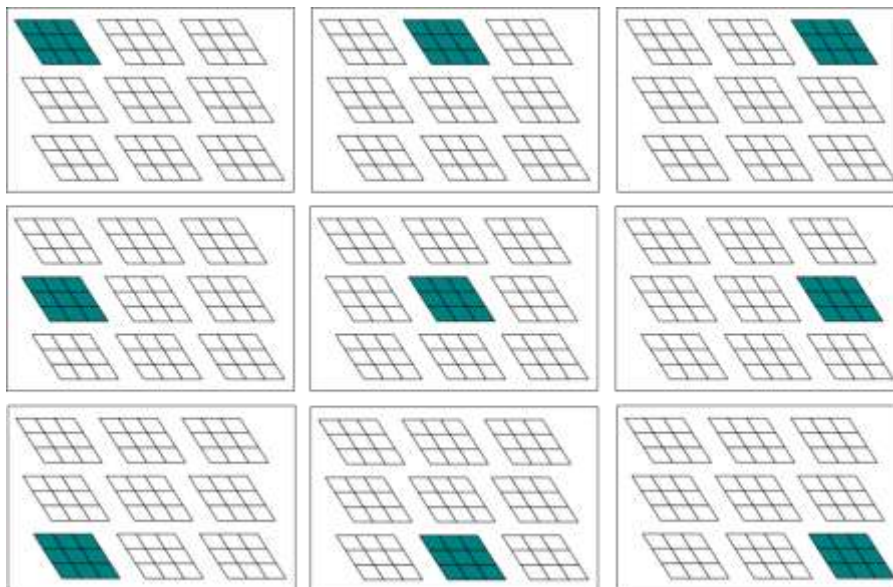
Your turn: On your turn, draw your symbol in any three spaces on the board not already occupied by a symbol. These three spaces do not need to be in a line.

The aim: Get nine of your symbol to form a plane of the 4D Noughts and Crosses board. The first player to do this wins. (A list of all the winning planes is given below.)

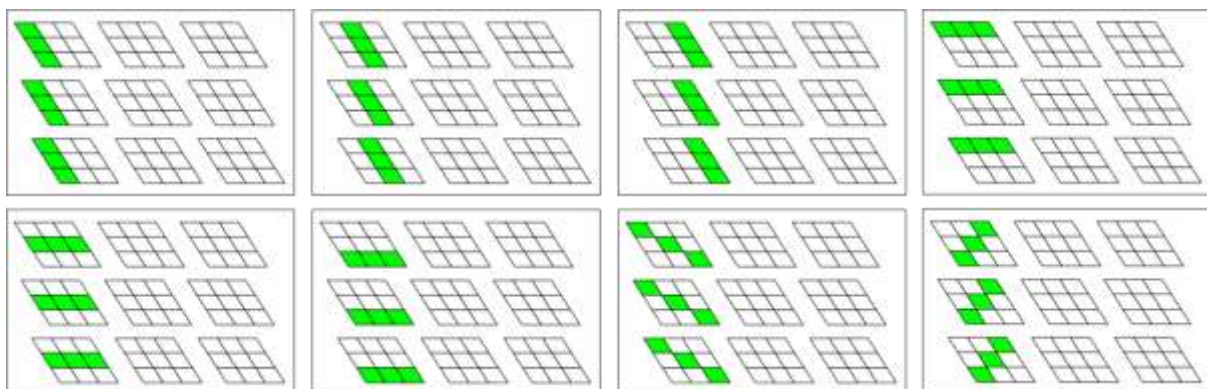
The winning planes

There are 154 planes in 4D Noughts and Crosses, all of which are shown below. They can be broken into five types:

1. A single flat:



2. Three lines, each in the same position in three flats in a row:

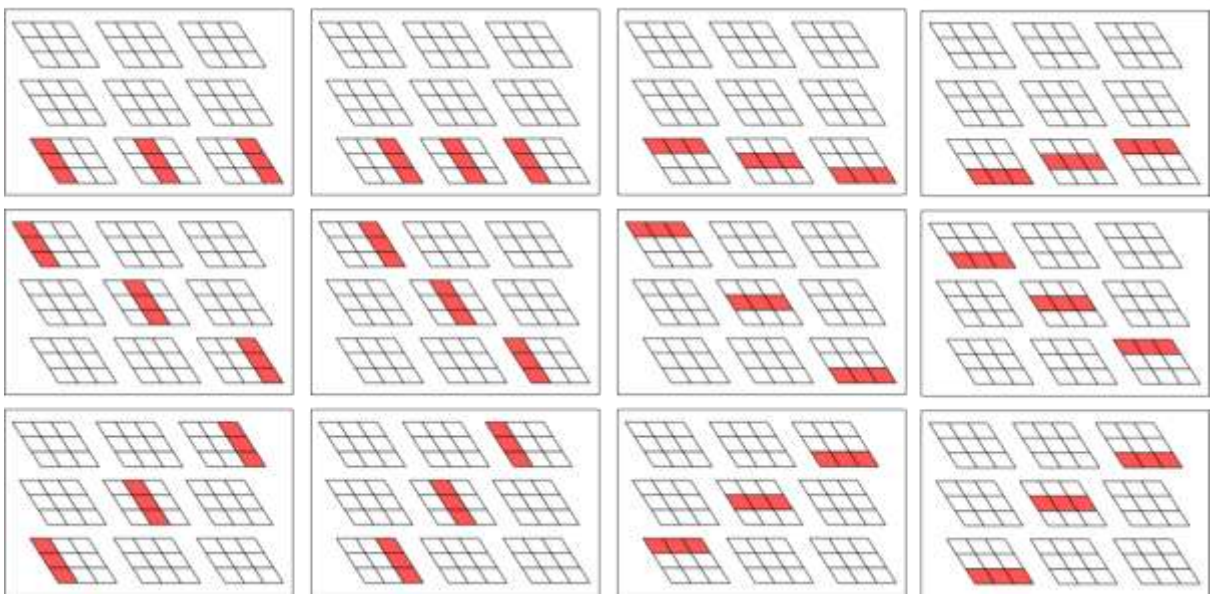




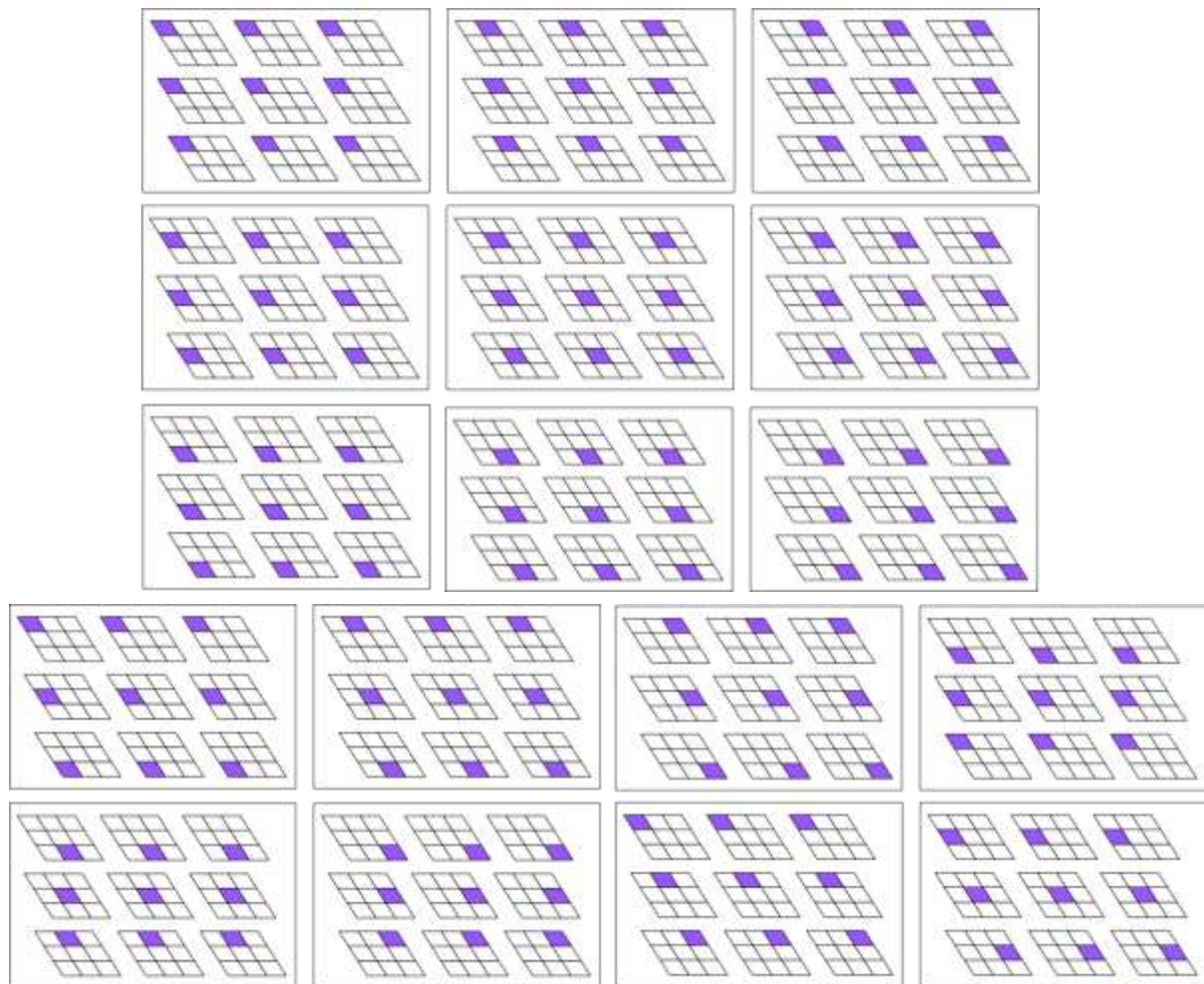


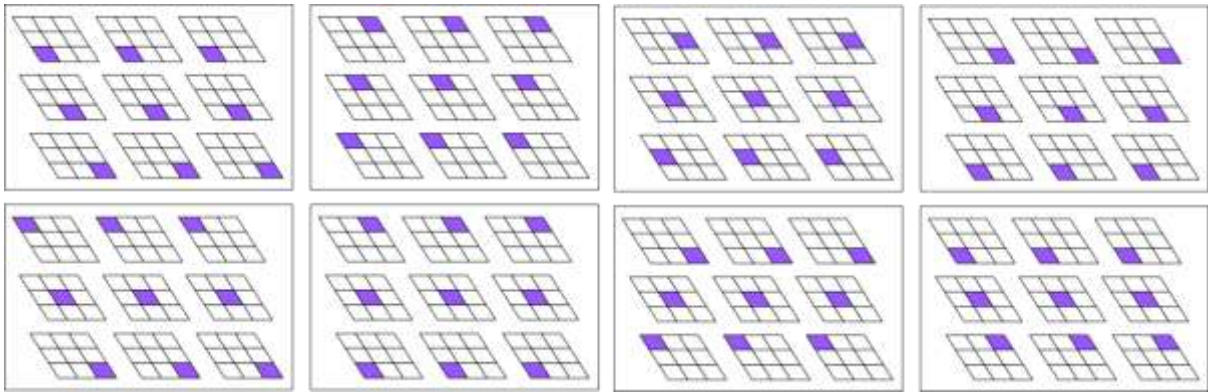
3. Three lines, each in a progressed position in three flats in a row:





4. Three vertical or diagonal lines, each in the same position in the three stacks:





5. Three vertical or diagonal lines, each in a progressed position in the three stacks:

