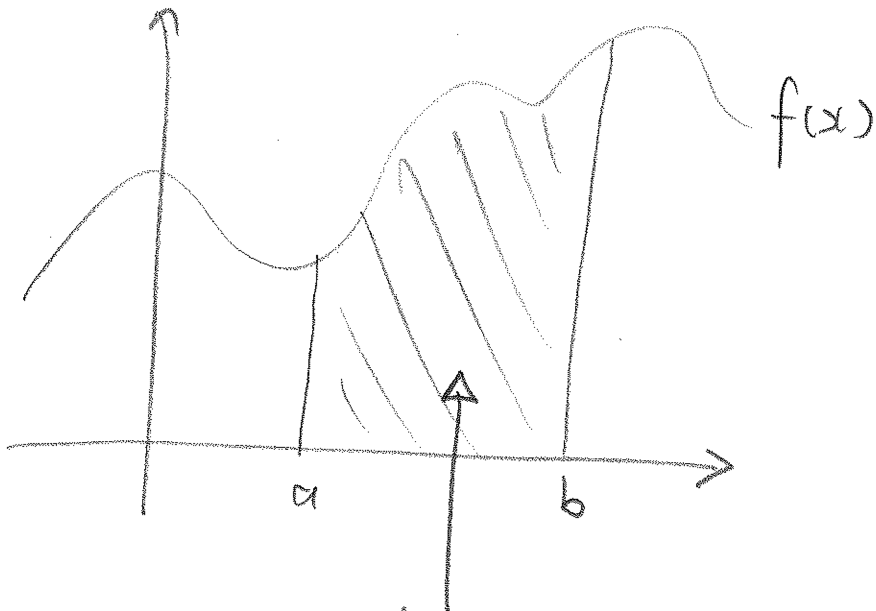
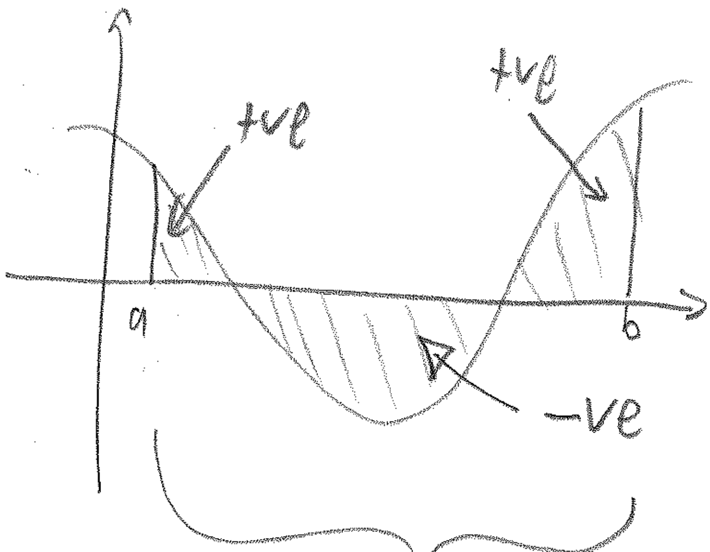
INTEGRATION

AQMF 2012

purpose: finding areas



$$\int_a^b f(x)\,dx$$



$$\int_a^b f(x)\,dx$$

Fund Theorem of Calc

If $F(x)$ is a function so that $F'(x) = f(x)$.

Then $\int_a^b f(x)\,dx = \left[F(x)\right]_a^b$

$= F(b) - F(a)$

which means:

integration is the opposite of differentiation

DEFINITE integral $\int_a^b f(x)\,dx$

↳ answer is a NUMBER

INDEFINITE integral $\int f(x)\,dx$

↳ answer is a FORMULA

with indefinite, trying to cover all possible functions that could diff to give $f(x)$.

↳ hence the "+C"

TECHNIQUES OF INTEGRATION

0. Just do it

eg $\int 4x^3 \, dx = 4 \times \frac{1}{4} x^4 + C$

$= x^4 + C$

power up by 1
divide by new power

eg $\int x^{-1} \, dx = \int \frac{1}{x} \, dx$

$= \ln x + C$

$$\boxed{\int \frac{1}{x} \, dx = \ln x + C}$$

eg $\boxed{\int e^x \, dx = e^x + c}$

eg $\int e^{4x} \, dx = \frac{1}{4} e^{4x} + c$

Divide by const. derivative

eg $\int \sqrt{3x+1} \, dx = \int (3x+1)^{\frac{1}{2}} \, dx$

$= \frac{2}{3}(3x+1)^{3/2} \times \frac{1}{3} + c$

$= \frac{2}{9}(3x+1)^{3/2} + c$

eg $\int \frac{1}{3-x} \, dx = \frac{1}{(-1)} \ln(3-x) + c$

$= -\ln(3-x) + c$

$\boxed{\text{1. Rewrite it}}$

eg $\displaystyle\int x^2(x+1)\,dx$

$$= \int (x^3 + x^2)\,dx$$

$$= \frac{1}{4}x^4 + \frac{1}{3}x^3 + C$$

eg $\displaystyle\int \frac{x+1}{\sqrt{x}}\,dx$

$$= \int (x^1+1)\times x^{-\frac{1}{2}}\,dx$$

$$= \int \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{1}\times x^{\frac{1}{2}} + C$$

$$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$$

2. Substitution

- for dealing with formulas inside other functions
- chain rule in reverse

eg $\int \boxed{x e^{(x^2)} \times dx}$

Let $\boxed{u = x^2}$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$\tfrac{1}{2}\,du = x\,dx$$

$$\int x e^{(x^2)}\,dx = \int e^{(x^2)} \times x\,dx$$

$$= \int e^{u} \times \tfrac{1}{2}\,du$$

$$= \int \tfrac{1}{2} e^{u}\,du$$

$$= \tfrac{1}{2} \times e^{u} + C$$

$$= \tfrac{1}{2} e^{(x^2)} + C$$

1. choose u (inside another function)

2. $\frac{du}{dx}$

3. rearrange to get ~~ du = ~~ dx

4. sub into integral until all x's are u's

5. do integral

6. sub in until all u's are x's.

eg $\displaystyle \int \boxed{\frac{(2x^3+3)}{(2x^4+12x)^5} \times \frac{dx}{1}}$ ([6(a) ass 4])

Let $u = 2x^4 + 12x$

$$\frac{du}{dx} = 2 \times 4x^3 + 12$$

$$= 8x^3 + 12$$

$$du = (8x^3 + 12)\,dx$$

$$\frac{1}{4}\,du = (2x^3 + 3)\,dx$$

$$\int \frac{(2x^3+3)\,dx}{(2x^4+12x)^5} = \int \frac{\left(\frac{1}{4}\right)\,du}{u^5}$$

$$= \int \frac{1}{4} u^{-5}\,du$$

$$= \frac{1}{4} \times \frac{1}{(-4)} u^{-4} + C$$

$$= -\frac{1}{16} u^{-4} + C$$

$$= -\frac{1}{16}(2x^4 + 12x)^{-4} + C$$

eg $\int \frac{\ln x}{x}\,dx$ [[Ass 10 Q6(a)]]

Let $u = \ln x$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x} \cdot dx$$

$$\int \frac{\ln x}{x}\,dx = \int \frac{\ln x}{1} \times \frac{1}{x}\,dx$$

$$= \int u\,du$$

$$= \frac{1}{2}u^2 + C$$

$$= \frac{1}{2}(\ln x)^2 + C$$

3. INTEGRATION BY PARTS

- for two things multiplied together
- will integrate one and diff. the other

CHOOSING WHICH IS WHICH

1. Integrate what you know how to integrate
2. Differentiate to make it go away.

differentiate integrate

$$\int uv'\,dx = uv - \int u'v\,dx$$

$$\int uv\,dx = uv - \int uv'dx$$

eg $\int x(x+3)^4\,dx$ [Ass 10 Q4(i)]

$u = x$ $\quad v = \frac{1}{5}(x+3)^5 \times \frac{1}{1}$ ← const derivative

$u' = 1$ $\quad v' = (x+3)^4$

$$\int x(x+3)^4\,dx$$

$$= \int u\,v'\,dx$$

$$= uv - \int u'v\,dx$$

$$= x \times \frac{1}{5}(x+3)^5 - \int 1 \times \frac{1}{5}(x+3)^5\,dx$$

$$= \frac{1}{5}x(x+3)^5 - \frac{1}{5} \times \frac{1}{6}(x+3)^6 \times \frac{1}{1} + C$$

$$= \frac{1}{5}x(x+3)^5 - \frac{1}{30}(x+3)^6 + C$$

eg $\int \frac{\ln x}{x}\,dx$ [[Ass 10 6(b)]]

$u = \ln x$ $\quad$ $v = \ln x$

$u' = \frac{1}{x}$ $\quad$ $v' = \frac{1}{x}$

$$\int \ln x \times \frac{1}{x}\,dx$$

$$= \int u\,v'\,dx$$

$$= uv - \int u'v\,dx$$

$= \ln x \times \ln x - \int \frac{1}{x} \times \ln x \, dx$

$\int \frac{\ln x}{x} \, dx = (\ln x)^2 - \int \frac{\ln x}{x} \, dx$

$2 \int \frac{\ln x}{x} \, dx = (\ln x)^2$

$\int \frac{\ln x}{x} \, dx = \frac{1}{2} (\ln x)^2 + C$

going round in circles

eg $\int x^2 e^{2x} \, dx$

Let $u = x^2$ $\quad v = \frac{1}{2} e^{2x}$

$u' = 2x$ $\quad v' = e^{2x}$

$\int x^2 e^{2x} \, dx$

$= \int u v' \, dx$

$= uv - \int u' v \, dx$

$= x^2 \times \frac{1}{2} e^{2x} - \int 2x \times \frac{1}{2} e^{2x} \, dx$

$$= \frac{1}{2}x^2e^{2x} - \int xe^{2x}\,dx$$

Redefine $u = x$ $\quad v = \frac{1}{2}e^{2x}$

$u' = 1$ $\quad v' = e^{2x}$

$$\int x e^{2x}\,dx$$

$$= \int u v'\,dx$$

$$= uv - \int u'v\,dx$$

$$= x \times \frac{1}{2}e^{2x} - \int 1 \times \frac{1}{2}e^{2x}\,dx$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2} \times \frac{1}{2}e^{2x}$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}$$

$$\int x^2e^{2x}\,dx = \frac{1}{2}x^2e^{2x} - \int xe^{2x}\,dx$$

$$= \frac{1}{2}x^2e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)$$

$$= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$