NEWTON'S METHOD
AND BISECTION METHOD

Finding approximations to zeros of functions.

Bisection Method

Advantage: Always works
Disadvantage: Takes a while

Newton's method

Advantage: Fast
Disadvantage: Sometimes doesn't work

BISECTION METHOD

Ex Find a zero of \( f(x) = \sin x - \frac{1}{2} x \)
between 1.5 and 2.
\[ f(1.5) = 0.247 \]
\[ f(2) = -0.091 \]
\[ x_1 = 1.75 \]
\[ f(1.75) = 0.1089 \]
\[ x_2 = 1.875 \]
\[ f(1.875) = 0.01165 \]
\[ x_3 = 1.9375 \]
\[ f(x_3) = -0.0252 \]
\[ x_4 = 1.90625 \]

So, the approx zero is 1.91 to 1 dp.

OK to round off \( x_i \)'s as you go to match level of accuracy.
NEWTON'S METHOD

Tangent and curve are similar close-up.
So calculate where TANGENT meets x-axis!

$(x_n, f(x_n))$

$(x_{n+1}, 0)$

$y = f(x)$

Current estimate

New estimate
Slope: \[ f'(x_n) = \frac{0 - f(x_n)}{x_{n+1} - x_n} \]

\[ x_{n+1} - x_n = \frac{-f(x_n)}{f'(x_n)} \]

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

**Example:** Find a zero of \( f(x) = \sin x - \frac{1}{2}x \) (other than \( x = 0 \)), to 2 d.p.

\[ f(x) = \sin x - \frac{1}{2} x \]
\[ f'(x) = \cos x - \frac{1}{2} \]

\[ x_{n+1} = x_n - \frac{\sin x_n - \frac{1}{2} x_n}{\cos x_n - \frac{1}{2}} \]

\( x_0 = 2 \)
\( x_1 = 2 - \frac{\sin (2) - \frac{1}{2} \times 2}{\cos (2) - \frac{1}{2}} \)
\( = 1.901 \) (3dp)

\( x_2 = 1.901 - \frac{\sin (1.901) - \frac{1}{2} \times 1.901}{\cos (1.901) - \frac{1}{2}} \)
\( = 1.896 \) (3dp)
\[ x_3 = 1.896 \quad (3\text{dp}) \]

So to 2dp the zero is

\[ x = 1.90 \]