**Vectors & Geometric Proofs**

**Commutativity**
\[ v + w = w + v \]

**Associativity**
\[ (v + w) + u = v + (w + u) \]

**Distributivity**
\[ (\alpha + \beta)v = \alpha v + \beta v \]
\[ k(v + w) = kv + kw \]

**Identity**
\[ 1 \cdot v = v \quad v + 0 = v \quad 0 \cdot v = 0 \]
\[ w + (-1)v = w - v \]

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Prove that the line segment joining the midpoint of two sides of a triangle is parallel to the third side.

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![Diagram](attachment:triangle.png)

Want to show \[ \overrightarrow{PQ} \text{ is parallel to } \overrightarrow{OB} \]

Parallel \( \iff \) one is a scalar multiple of the other

\[ \overrightarrow{PQ} = k \cdot \overrightarrow{OB} \]

\( P \) is midpoint of \( OC \): \[ OP = \frac{1}{2} OC \]

\( OB = OC + CB \)

\( OQ = OC + \frac{1}{2} CB \)

\( OQ \) is the midpoint of \( CB \)

\[ PQ = OQ - OP = OC + \frac{1}{2} CB - \frac{1}{2} OC \]

\[ = OC - \frac{1}{2} OC + \frac{1}{2} CB \]
\[ PQ = \frac{1}{2} OC + \frac{1}{2} CB \]
\[ = \frac{1}{2} (OC + CB) = \frac{1}{2} (OB) \]

\[ \therefore PQ \text{ is parallel to } OB \text{ as required.} \]

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Find the plane through these points:
(0, 0, 2) \quad (0, 3, 0) \quad (5, 0, 0) \quad P = (0, 0, 2) \quad Q = (0, 3, 0) \quad R = (5, 0, 0) \quad PQ = Q - P = (0, 3, -2) \quad PR = R - P = (5, 0, -2)

\[
PQ \times PR = \begin{vmatrix} 0 & 3 & -2 \\ 5 & 0 & -2 \\ 1 & 1 & 1 \end{vmatrix}
\]

\[
\begin{bmatrix} 3 & -2 \\ 0 & -2 \\ 1 & 1 \end{bmatrix}
\]

\[ n = (-6, -10, -15) \]

\[
\begin{align*}
-6x - 10y - 15z &= d \\
-6(1) - 10(2) - 15(0) &= d \\
-26 &= d
\end{align*}
\]

\[ n' = (6, 10, 15) \]

\[
6x + 10y + 15z = d'
\]
find the equation of the plane through $n = (1, 2, 3) + \text{perpendicular to } l = (1, 0, 1) + t (-1, 2, 7)$ as $l$ is perpendicular to the plane the direction of the line is the normal direction of $l$ is $(-1, 2, 7)$

$n \cdot (x, y, z) = n \cdot p$

$(-1, 2, 7) \cdot (x, y, z) = (-1, 2, 7) \cdot (1, 2, 3)$

$-x + 2y + 7z = -1 + 4 + 21 = 24$

$-x + 2y + 7z = 24.$