Logic & Proofs

Most maths makes statements about objects

\[ 2n + 1 \text{ is an odd number}\]

\[ \begin{bmatrix} \vec{e}_x, \vec{e}_y \end{bmatrix} \text{ is a unit vector} \]

Mathematical logic makes statements about statements

\[ P \Rightarrow Q \text{ is equivalent to } \neg Q \Rightarrow \neg P \]

Most interested in whether the "flow of logic" is valid.
Things to do to statements

* The contrapositive

\[ P \implies Q \]

the contrapositive is

\[ \neg Q \implies \neg P \]

eg "If you are human then you have eyes."

CONTRAPOSITIVE

eg "If you don't have eyes then you are not human."

eg "If \( n \) is even then so is \( n^2 \)"

\[ P \]

CONTRAPOSITIVE

eg "If \( n^2 \) is not even then \( n \) is not even.\]
"If $A$ is invertible\n\[P\] \quad \Rightarrow \quad \text{det}(A) \neq 0\n\[Q\]
**Contrapositive**

"If $\text{det}(A) = 0\n\[\neg Q\]
then $A$ is not invertible\n\[\neg P\]

The contrapositive of any statement is logically equivalent to the original.

* The converse

\[P \Rightarrow Q\]
the converse is
\[Q \Rightarrow P\]
eg: "If you are human then you have eyes"

CONVERSE

"If you have eyes then you are human"

The converse of a statement is logically independent from the original statement.

* Negation

eg: "I am human"
NEGATIVE

"I am not human"
eg: "Every person is human"

or "A person, person is human"

NEGATIVE

"Not all people are human"

or "There is a person who is not human"

or "\( \exists \text{ person, person not human} \)"

eg "\( \forall x \in \mathbb{N}, 2x+1 \) is odd"

NEGATIVE

"\( \exists x \in \mathbb{N}, 2x+1 \) is even"
The opposite of "Everyone is here" is not "No-one is here".

In fact, the opposite of "Everyone is here" is "Someone is not here".
If there is a prime number $p$ such that $2^p + 1$ is prime then Fermat's theorem is false.

CONTRAPOSITIVE

If Fermat's theorem is true then

every prime number $p$ is such that $2^p + 1$ is not prime.

The negative of a statement is not equivalent to the original.
If all cats speak French and some chickens are cats, then some chickens speak French.