RELATED RATES

Situation where things change

Goal is to know rate of change (e.g. speed) at some moment.

Why hard is what the different bits represent and how they're related.

Ex 4.4.6 p144 maths in notes.

Rate of change of area

Thickness same everywhere

Volume fixed

Rate of change of thickness
Choose variables:

\[ \text{radius} \quad \text{area} \quad \text{volume} \quad \text{thickness} \quad \text{time} \]

\[ r \quad A \quad V \quad d \quad t \]

Formulas:

\[ A = \pi r^2 \]

\[ V = A d = \pi r^2 d \]

Derivatives with time:

\[ A = \pi r^2 \]

\[ \frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} \quad (1) \]

\[ V = A d \]

\[ \frac{dV}{dt} = \frac{dA}{dt} \cdot d + A \cdot \frac{dd}{dt} \quad (2) \]

When \( r = 200 \text{ m}, \quad \frac{dr}{dt} = 0.1 \text{ m/min} \)

Eqn(1):

\[ \frac{dA}{dt} = \pi \times 2 \times 200 \times 0.1 \]

\[ = \pi \times 40 \]

\[ = 125.7 \text{ m}^2/\text{min} \quad (1 \text{ dp}) \]
Also \( r = 200 \), \( \frac{dA}{dt} = 50 \pi \), \( d = 0.01 \)

\[ A = \pi r^2 = \pi \times 200^2 \]

\[ \frac{dV}{dt} = 0 \quad \text{(Volume fixed.)} \]

Eqn 2: \[ 0 = 40 \pi \times 0.01 + 200^2 \pi \cdot \frac{dp}{dt} \]

\[ 200^2 \pi \frac{dp}{dt} = -40 \pi \times 0.01 \]

\[ \frac{dp}{dt} = \frac{-40 \pi \times 0.01}{200^2 \pi} \]

\[ = -\frac{40 \times 0.01}{20000} \]

\[ = -0.00001 \text{ min}^{-1} \]

---

(from faculty of math at Illinois Uni)

50 ft ladder against a large building.

Base slips at rate 3 ft per min.

Find rate of change of height of ladder top above ground at instant when bottom is 20 ft from building.
Let $x = \text{dist (ft) of bottom from building}$.

$y = \text{dist (ft) of top from ground}$.

Pythagoras: $x^2 + y^2 = 50^2$

Derivative (t): $2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

$\frac{x \cdot dx}{dt} + y \cdot \frac{dy}{dt} = 0$

When $x = 30 \text{ ft}$

$\frac{dx}{dt} = 3 \text{ ft/min}$

Find $y$: $x^2 + y^2 = 50^2$

$y = \sqrt{50^2 - x^2} = \sqrt{50^2 - 30^2} = 40$
Sub in:

\[ 80 \times 3 + 40 \times \frac{dy}{dt} = 0 \]

\[ 40 \times \frac{dy}{dt} = -3 \times 80 \]

\[ \frac{dy}{dt} = -\frac{3 \times 80}{40} \]

\[ = -2.25 \text{ ft/min} \]

So top is sliding down at 2.25 ft/min.