UniSTEP / MLC Seminars:

Maths in Lectures: Understanding the Notation

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Where you'll see maths notation

- Maths (duh!)
- Statistics
- Physics
- Chemistry
- Economics
- Psychology
- Almost any discipline at all

Why people use maths notation

Because ***it makes life easier!***

- Easier to write maths down
- Easier to be accurate
- Easier to communicate with other languages
- Easier to think

How to understand maths notation

Ask yourself:

- How do you **say** it?
- What does it **mean**?
- What are the **rules** for working with it?
- How is it **connected** to other ideas?

Example: $\sqrt{}$

- How do you **say** it?

  $\sqrt{25}$ – "The square root of 25"

  "Root 25"

- What does it **mean**?

  $\sqrt{x}$ is the number you square to get x.

  For example, $\sqrt{25} = 5$ because $25 = 5^2$.

Example: $\sqrt{\quad}$

- What are the **rules** for working with it?
  - *Can* distribute it over multiplication and division:

$$\sqrt{4\times 100}=\sqrt{4}\times\sqrt{100} \qquad \sqrt{\frac{3}{19}}=\frac{\sqrt{3}}{\sqrt{19}}$$

  - *Can't* distribute it over addition and subtraction:

$$\sqrt{25+16}\ \text{ IS }\mathbf{NOT}\ \sqrt{25}+\sqrt{16}$$

  - *Square* a number if you bring it inside:

$$3\sqrt{2}=\sqrt{9\times 2}$$

Example: $\sqrt{}$

- How is it **connected** to other ideas?
  - The opposite of squaring
  - $\sqrt{x}$ can also be written as $(x)^{\frac{1}{2}}$
  - Use it to find distances
  - Use it to find the standard deviation
  - Used it to solve quadratic equations
  - Similar rules to $\sqrt[3]{}, \sqrt[4]{}, \sqrt[5]{}, \ldots$

Where to find these answers

- Listen to your teachers as they write
- Look for definitions nearby in the notes/book
- Notice the rules in written examples
- Ask someone

like the Maths Learning Centre
Level 3 East, Hub Central
10am to 4pm weekdays

Types of notation

- Notation for **naming** things
- Notation for **making statements** about things
- Notation for **creating things** from old things
- Notation for **abbreviating** words and phrases

Notation for ***naming***

Often need to name something you're talking about. For example "Let x be the number we want to find..."

- Greek letters
- Well-known objects
- Vectors
- Subscripts
- Distributions

Naming: Greek Letters

Α α - alpha
Β β - beta
Γ γ - gamma
Δ δ - delta
Ε ε - epsilon
Ζ ζ - zeta
Η η - eta
Θ θ - theta

Ι ι - iota
Κ κ - kappa
Λ λ - lambda
Μ μ - mu
Ν ν - nu
Ξ ξ - xi
Ο ο - omicron
Π π - pi

Ρ ρ - rho
Σ σ - sigma
Τ τ - tau
ϒ υ -upsilon
Φ φ - phi
Ψ ψ - psi
Χ χ - chi
Ω ω - omega

Naming: Greek Letters

α - alpha	ι - iota	ρ - rho
β - beta	κ - kappa	Σ σ - sigma
Γ γ - gamma	Λ λ - lambda	τ - tau
Δ δ - delta	μ - mu	ϒ υ -upsilon
ε - epsilon	ν - nu	Φ ϕ - phi
ζ - zeta	Ξ ξ - xi	Ψ ψ - psi
η - eta		χ - chi
Θ θ - theta	Π π - pi	Ω ω - omega

Naming: Well-known objects

e – e is approximately 2.71828...

π – pi is approximately 3.14159...

∞ – infinity

$\varnothing$ – the empty set

$\mathbb{N}$, **N** – the set of natural numbers

$\mathbb{Z}$, **Z** – the set of integers

$\mathbb{Q}$, **Q** – the set of rational numbers

$\mathbb{R}$, **R** – the set of real numbers

$\mathbb{C}$, **C** – the set of complex numbers

Naming: Vectors

In print, vectors are usually written in **bold**:

$$\mathbf{u} \quad 3\mathbf{v} \quad \mathbf{e}$$

In handwriting, they have an extra mark:

$$\bar{v} \quad \vec{v} \quad \tilde{v} \quad \underline{v} \quad \underset{\rightarrow}{v} \quad \underset{\sim}{v}$$

Please *mark* your vectors:

GOOD $a\underline{v} + b\underline{u}$

BAD $av + bu$

Naming: Subscripts

Subscripts help to give names to related things (don’t say it’s a subscript when you read it aloud):

$$c_1, c_2, c_3, c_4, c_5 \qquad \mathbf{v} = (v_1, v_2, v_3)$$

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \qquad \mathbf{e}_r, \mathbf{e}_n$$

People use an “i” to refer to all of them at once:

$$c_i \text{ for } i = 1, 2, 3, 4, 5$$

Naming: Distributions

The letters tell you which family of distribution and the numbers tell which one in that family.

$N(28,3)$ – Normal distribution with mean 28 and standard deviation 3

t_{14} – t distribution with 14 degrees of freedom

χ^2_5 – chi-squared distribution with 5 degrees of freedom

$F(2,30)$ – F distribution with 2 numerator and 30 denominator degrees of freedom

$B(10,0.7)$ – Binomial distribution with n = 10 and p = 0.7

Notation for ***<u>making statements</u>***

These notations go between bits of maths to make a statement.

Read them aloud differently depending on context:

Let x = 6. Then x = 1+5 = 1+2+3.

"Let x be equal to 6. Then x is equal to 1 plus 5, which is equal to 1 plus 2 plus 3."

Statements: about numbers

$\leq$ – "is less than or equal to"

$<$ – "is less than"

$\geq$ – "is greater than or equal to"

$>$ – "is greater than"

$=$ – "is equal to"

$\neq$ – "is not equal to"

$\approx$, $\doteqdot$, $\simeq$ – "is approximately equal to"

$\propto$ – "is proportional to"

$\equiv$ – "is equivalent to"

Statements: about sets

for two sets
- $\subset$ – "is contained in", "is a subset of"
- $\subseteq$ – "is contained in or equal to"

For example:

$\mathbf{N} \subset \mathbf{R}$ – "The set of natural numbers is contained in the set of real numbers"

for an object and a set
- $\in$ – "is in", "is an element of"
- $\notin$ – "is not in", "is not an element of"

For example:

$e \notin \mathbf{Q}$ – "e is not in the set of rational numbers"

Statements: about other things

for lines
- $\perp$ – "is perpendicular to"
- $\parallel$ – "is parallel to"

for a random variable
- $\sim$ – "has the ___ distribution"

For example:

$X \sim \chi^2_5$ – "X has the chi-squared distribution with 5 degrees of freedom"

for abstract algebraic objects
- $\cong$ – "is isomorphic to"

Notation for ***creating***

Some notations are for making new objects/numbers from old ones.

- Binary operations
- Symbols that work on one number
- Functions
- Complicated things

Creating: Operations on numbers

$5+4$ – "5 plus 4"

$5-4$ – "5 minus 4"

5×4 – "5 times 4", "5 multiplied by 4"

$5\cdot 4$ – "5 times 4", "5 multiplied by 4"

xy – "x times y", "xy"

$5\div 4$ – "5 divided by 4"

$5/4$ – "5 divided by 4", "5 over 4"

5^4 – "5 to the power of 4"

5^2 – "5 squared", "5 to the power of 2"

5^3 – "5 cubed", "5 to the power of 3"

Interlude: The Order of Operations

Operations are done in a certain order:

$(\ ), [\ ], \{\ \}$	1. Anything in brackets
x^2	2. Powers
$\div, \times$	3. Division and Multiplication
$-, +$	4. Subtraction and Addition

$$4(5+6)-\frac{4+14}{2\times 3}+3\div 6\times 7-(3+4[8-2])$$

Interlude: The Order of Operations

$$4(5+6)-\frac{4+14}{2\times 3}+3\div 6\times 7-(3+4[8-2])$$

$$=4(5+6)-\frac{4+14}{2\times 3}+3\div 6\times 7-(3+4\times 6)$$

$$=4(5+6)-\frac{4+14}{2\times 3}+3\div 6\times 7-(3+24)$$

$$=4\times 11-\frac{18}{6}+3\div 6\times 7-(27)$$

$$=4\times 11-3+\tfrac{1}{2}\times 7-27$$

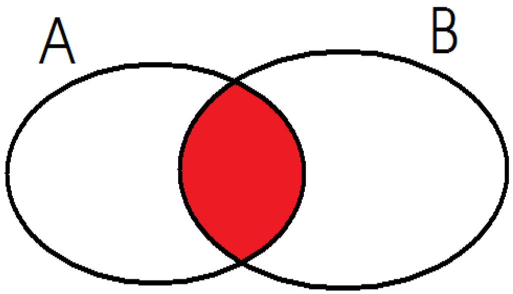
$$=44-3+3\tfrac{1}{2}-27$$

$$=27\tfrac{1}{2}$$

Creating: Operations on sets

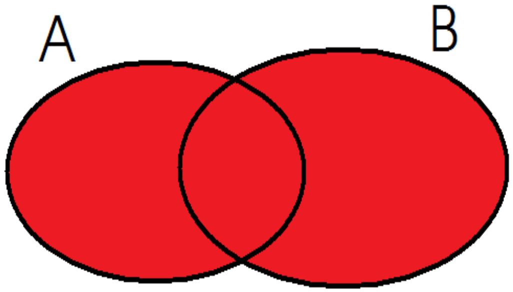
$A \bigcap B$ – "A intersection B", "the intersection of A and B"

– the set of all the things in both A and B



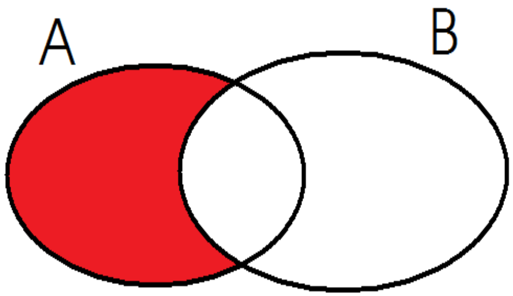
$A \bigcup B$ – "A union B", "the union of A and B"

– the set of all the things in either A or B



$A \setminus B$ – "A without B", "the exclusion of B from A"

– the set of all the things in A but not B



Creating: Symbols for one number

$\sqrt{x}$ – "the square root of 25"

– the number you square to get 25

$\sqrt[3]{x}$ – "the cube root of 25"

– the number you cube to get 25

$\sqrt[4]{x}$ – "the fourth root of 25"

$|x|$ – "the absolute value of x", "mod x"

– if x is negative, make it positive

$5!$ – "5 factorial"

– the product of the numbers up to 5: $1\times2\times3\times4\times5$

Creating: Functions

All of these usually refer to the *answer* produced by the function, which is a new number.

$f(x)$ – "f of x"

– NOT f multiplied by x!

$\sin x$ – "sine x", "sine of x"

$\cos x$ – "cos x", "cos of x"

$\tan x$ – "tan x", "tan of x"

} trigonometric

Creating: Functions

All of these usually refer to the *answer* produced by the function, which is a new number.

$\ln x$ – "Ell-En x", "Ell-En of x"

– the natural logarithm of x: if you do $e^{\text{this number}}$ you get x as your answer

– some people write this as $\log x$

$\log_{10} x$ – "log base 10 of x", "log 10 of x"

– the base 10 logarithm of x: if you do $10^{\text{this number}}$ you get x as your answer

– some people write this as $\log x$

Creating: Sets

$\{x \in \mathbb{R} \mid x > 1\}$ – "the set of x which are in the real numbers such that x is greater than 1"

$\{a^2 + 1 \mid a \in \mathbb{R}\}$ – "the set of numbers a squared plus 1 such that a is in the real numbers."

$\left\{1, 3, \pi, \sqrt{2}\right\}$ – "the set containing, 1, 3, pi and the square root of 2"

Creating: Sets - Intervals

(1,5) – “the set of numbers between 1 (not including 1) and 5 (not including 5)”

(1,5] – “the set of numbers between 1 (not including 1) and 5 (including 5)”

[1,5] – “the set of numbers between 1 (including 1) and 5 (including 5)”

(1,∞) – “the set of numbers from 1 (not including 1) upwards”

(- ∞,5] – “the set of numbers from 5 (including 5) downwards”

Creating: Complicated things

$\int_0^5 x^2 + 3x\,\mathrm{d}x$ – "the integral from 0 to 5 of x squared plus 3 x d x"

$\sum_{i=1}^{7}(i^2+2)$ – "the sum of i squared plus 2, as i ranges from 1 to 7"

$\left.\frac{dy}{dx}\right|_{x=3}$ – "dy on dx evaluated when x is equal to 3"

$\lim_{x\to\infty}\frac{1}{x}$ – "the limit, as x approaches infinity, of 1 over x"

Notation for ***abbreviating***

Shortcuts for writing things because mathematicians are lazy or want to talk to people in other countries.

Abbreviating

$x \to 3$ – "x approaches 3"

$f : \mathbf{R} \to \mathbf{R}$ – "the function f sends the real numbers to the real numbers"

$\Rightarrow$ – "implies that"

$\Leftrightarrow$, iff – "if and only if"

wrt – "with respect to"

st – "such that"

$\forall$ – "for all", "for every"

$\exists$ – "there exists"

$\exists!$ – "there exists a unique"

THE END

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Level 3 East, Hub Central

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