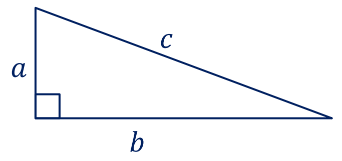
2D and 3D Trigonometry Facts

By the Maths Learning Centre, University of Adelaide

Pythagoras

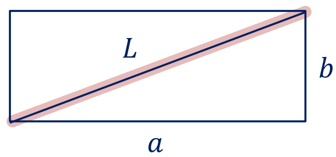
In a right-angled triangle, if the short sides are a and b, and the long side is c, then

$$c^2 = a^2 + b^2.$$



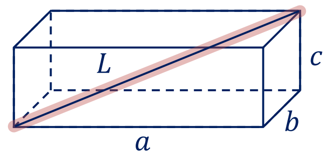
$$c^2 = a^2 + b^2$$

In 2D, if a rectangular box has sides a and b, then the diagonal is $\sqrt{a^2 + b^2}$.



$$L = \sqrt{a^2 + b^2}$$

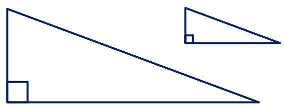
In 3D, if a rectangular box has edges a, b and c, then the diagonal is $\sqrt{a^2 + b^2 + c^2}$.

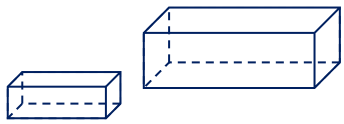


$$L = \sqrt{a^2 + b^2 + c^2}$$

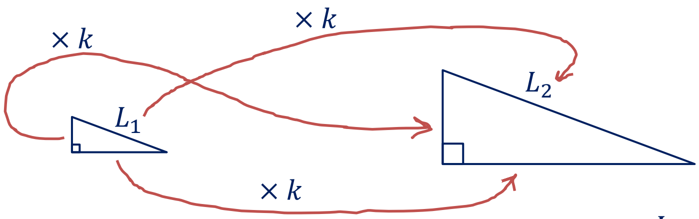
Similar triangles (and other objects)

Two objects are called **similar** when they are exactly the same shape with the same angles (though they may possibly be different sizes).





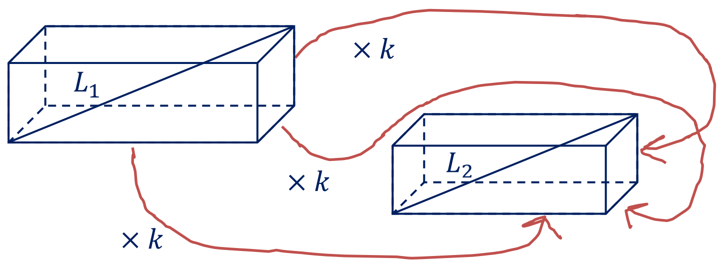
Multiplying or dividing all the lengths of an object by the same constant (but keeping all the angles the same) produces an object that is similar.



$$k = \frac{L_2}{L_1}$$

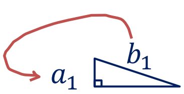
If two objects *are* similar, then there is a constant k so that every length in one is k times the matching length in the other.

If a length L_1 in the first object matches with a length L_2 in the second object then this constant k is $\frac{L_2}{L_1}$.

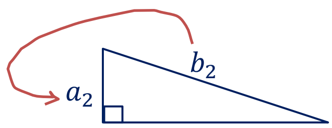


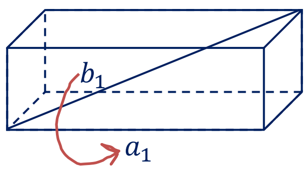
If two objects are similar, then matching pairs of lengths are in the same ratio.
That is, if length a_1 in the first object matches with length a_2 in the second object, and length b_1 in the first object matches with length b_2 in the second object, then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}.$$

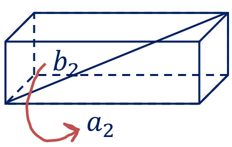


$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$





$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$



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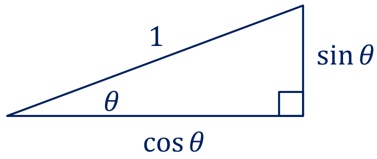
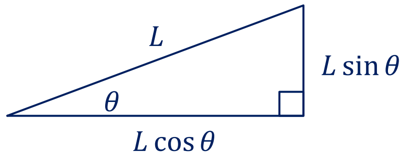
Right-angled triangles

If the hypotenuse of a right-angled triangle is 1 then the side opposite the angle θ is $\sin\theta$, and the side adjacent to the angle θ is $\cos\theta$.

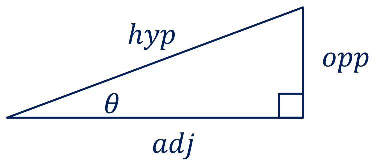


If the hypotenuse of a right-angled triangle is L then the side opposite the angle θ is $L\sin\theta$, and the side adjacent to the angle θ is $L\cos\theta$.



If one angle in a right-angled triangle is θ, and also the hypotenuse is "*hyp*", the side adjacent to θ is "*adj*" and the side opposite to θ is "*opp*", then

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$



$$\sin\theta = \frac{opp}{hyp}$$

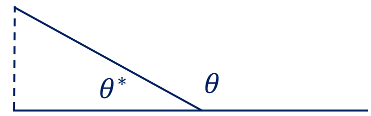
$$\cos\theta = \frac{adj}{hyp}$$

$$\tan\theta = \frac{opp}{adj}$$

Other angles and triangles

For an angle θ, the supplementary angle is the angle θ^* such that $\theta + \theta^* = 180°$.
If θ is between 90° and 180°, then the values of $\cos\theta$ and $\sin\theta$ are calculated using the supplementary angle as follows:
$\cos\theta = -\cos\theta^*$; $\sin\theta = \sin\theta^*$.

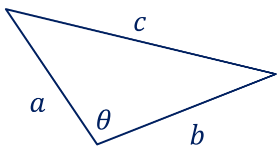
$\sin\theta = \sin\theta^*$



$\cos\theta = -\cos\theta^*$

The cosine rule:
If the sides of a triangle are a, b, c and the angle *opposite* c is θ, then

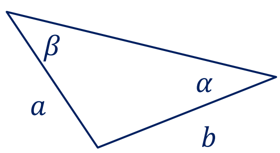
$$c^2 = a^2 + b^2 - 2ab\cos\theta\,.$$



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

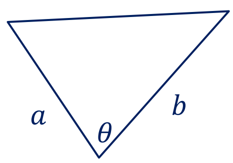
The sine rule:
In a triangle, if side a is opposite angle α and side b is opposite angle β , then

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}.$$



$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

If two sides of a triangle are a and b and the angle *between* them is θ, then the area of the triangle is $\frac{1}{2}ab\sin\theta$.



$$\text{Area} = \frac{1}{2}ab\sin\theta$$