Maths Learning Service: Revision ***Mathematics IA***

Index Laws

Mathematics IMA

Intro. to Fin. Maths I

Index laws are the rules for simplifying expressions involving *powers* of the same *base* number.

$a^m \times a^n = a^{m+n}$	First Index Law
$(a^m)^n = a^{mn}$	Second Index Law
$\dfrac{a^m}{a^n} = a^{m-n}$	Third Index Law
$a^{-m} = \dfrac{1}{a^m}$	
$a^0 = 1$	
$a^{\frac{1}{n}} = \sqrt[n]{a}$	

Examples: Simplify the following expressions, leaving only positive indices in the answer.

(a) $$\frac{3^6 2^4}{3^4} = \frac{3^6}{3^4} \times 2^4 = 3^2 2^4$$

(b) $$3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

(c) $$\frac{9\left(x^2\right)^3}{3xy^2} = \frac{9}{3} \times \frac{x^6}{x} \times \frac{1}{y^2} = 3 \times x^5 \times \frac{1}{y^2} = \frac{3x^5}{y^2}$$

(d) $$a^{-1}\sqrt{a} = a^{-1}a^{\frac{1}{2}} = a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}} \text{ or } \frac{1}{\sqrt{a}}$$

Notes: (1) More involved fractional powers can be dealt with by noting that $\boxed{a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m}$ by the Second Index Law. For example,

$$(27)^{\frac{2}{3}} = (27^{\frac{1}{3}})^2 = (\sqrt[3]{27})^2 = (3)^2 = 9.$$

(2) Watch out for powers of negative numbers. For example,

$$(-2)^3 = -8 \text{ and } (-2)^4 = 16, \text{ so } (-x)^5 = -x^5 \text{ and } (-x)^6 = x^6.$$

(3) In general $\boxed{(ab)^n = a^n b^n}$. For example,

$$(3x^2y)^3 = 3^3(x^2)^3y^3 = 27x^6y^3.$$

Exercises

1. Simplify the following expressions, leaving only positive indices in the answer.

(a) $4^2 \times 4^{-3}$ (b) $\dfrac{3^2\left(2^2\right)^{-2}}{2^3}$ (c) x^5x^8

(d) $(y^4)^6$ (e) $(-3)^3$ (f) $(4ab^2c)^3$

(g) $x^2z^{-3} \times (xz^2)^2$ (h) $2^n \times (2^{-n})^3 \times 2^{2n}$ (i) $3^m \times 27^m \times 9^{-m}$

(j) $(a^{\frac{1}{2}} \times a)^5$ (k) $\dfrac{(-2ab)^2}{2b}$ (l) $\dfrac{(-a^4b)^3(ab)^5}{-a^8b^8}$

(m) $\dfrac{x^{-1}y^4}{x^{-5}y^{-3}}$ (n) $\left(\dfrac{10a^3b^{-2}}{5a^{-1}b^2}\right)^{-1}$ (o) $x\sqrt[3]{x}$

(p) $(a^2 \times \sqrt{a})^2$ (q) $\dfrac{2x^{\frac{1}{2}}x}{x^2}$ (r) $(3a)^{-1} \times 3a^{-1}$

(s) $32^{\frac{3}{5}}$ (t) $\left(\dfrac{4}{25}\right)^{\frac{3}{2}}$ (u) $\left(4^{\frac{1}{3}}\right)\left(2^{\frac{1}{3}}\right)$

Terms involving the "$\sqrt{\ }$" symbol are known as a **radicals** or *surds*.

Notes: (1) $\boxed{\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}}$. For example $\sqrt{144+25} = \sqrt{169} = 13$

but $\sqrt{144} + \sqrt{25} = 12 + 5 = 17$.

(2) Similarly, $\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$.

(3) $\boxed{\sqrt{ab} = \sqrt{a} \times \sqrt{b}}$. For example $\sqrt{4 \times 9} = \sqrt{36} = 6$ and $\sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$.

(4) $\boxed{\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}}$. For example $\sqrt{\dfrac{16}{4}} = \sqrt{4} = 2$ and $\dfrac{\sqrt{16}}{\sqrt{4}} = \dfrac{4}{2} = 2$.

These techniques can be used to simplify radicals. For example

$$\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}\,.$$
$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}\,.$$

When asked to simplify radical expressions involving fractions, you are required to produce a single fraction (as in ordinary algebra) *with no radicals in the denominator.* For example

$$
\begin{aligned}
\frac{\sqrt{3}}{\sqrt{2}} + \frac{2}{\sqrt{6}} &= \frac{\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{2}} + \frac{2}{\sqrt{6}} \\
&= \frac{3}{\sqrt{6}} + \frac{2}{\sqrt{6}} \\
&= \frac{5}{\sqrt{6}} \\
&= \frac{5}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} \\
&= \frac{5\sqrt{6}}{6}
\end{aligned}
$$

Exercises (continued)

2. Simplify the following expressions

(a) $\sqrt{50}$ (b) $\sqrt{72}$ (c) $\sqrt{12} + \sqrt{27}$

(d) $\dfrac{1}{\sqrt{5}} - \dfrac{1}{\sqrt{10}}$ (e) $\dfrac{\sqrt{2}}{\sqrt{3}} + \dfrac{\sqrt{3}}{\sqrt{2}}$ (f) $\dfrac{1}{\sqrt{3}} - \dfrac{2\sqrt{3}}{\sqrt{15}}$

Answers to Exercises

1. (a) $\dfrac{1}{4}$ (b) $\dfrac{9}{2^7} = \dfrac{9}{127}$ (c) x^{13} (d) y^{24} (e) -27
   (f) $64a^3b^6c^3$ (g) x^4z (h) 1 (i) 3^{2m} (j) $a^{15/2}$
   (k) $2a^2b$ (l) a^9 (m) x^4y^7 (n) $\frac{1}{2}\,a^{-4}b^4$ (o) $x^{4/3}$
   (p) a^5 (q) $2x^{-1/2}$ (r) a^{-2} (s) 8 (t) $\dfrac{8}{125}$
   (u) 2

2. (a) $5\sqrt{2}$ (b) $6\sqrt{2}$ (c) $5\sqrt{3}$ (d) $\dfrac{2\sqrt{5}-\sqrt{10}}{10}$
   (e) $\dfrac{5\sqrt{6}}{6}$ (f) $\dfrac{5\sqrt{3}-6\sqrt{5}}{15}$