Sample Cheat sheet for AQMF

S-Sample space, E an event in, or subset of S, $Pr(E)=\frac{n(E)}{n(S)}$. If $Pr(E)=1$ then E is certain to occur, if $Pr(E)=0$ then E is impossible. Intersection of E and F, $E\cap F=\{x\in S|x\in E \text{ and } x\in F\}$. Union of E and F, $E\cup F=\{x\in S|x\in E \text{ or } x\in F\}$. The complement of E, $E^c=\{x\in S|x\notin E\}$. $E\cap E^c=\phi$ (the empty set) and $E\cup E^c=S$. So $Pr(E^c)=1-Pr(E)$. Inclusion-exclusion principle $Pr(E\cup F)=Pr(E)+Pr(F)-Pr(E\cap F)$. Events E and F are mutually exclusive if $E\cap F=\phi$. prob of A given B: $Pr(A|B)=\frac{Pr(A\cap B)}{Pr(B)}$ or $Pr(A\cap B)=Pr(A|B)Pr(B)=Pr(B|A)Pr(A)$. prob trees, the prob of one path is the product of probabilities on all of the branches along the path. If an event can be described by more than one path, then the prob of the event is the sum of the prob for each path. **Bayes formula** $Pr(B|A)=\frac{Pr(A|B)Pr(B)}{Pr(A)}$. Events A and B are independent if $Pr(A|B)=Pr(A)$ and $Pr(B|A)=Pr(B)$. Events $B_1,B_2,\ldots B_n$ are: exhaustive if $B_1\cup B_2\cup\ldots\cup B_n=S$,
disjoint if $B_i\cap B_j=\phi$ if $i\neq j$
and if both, then $Pr(A)=Pr(A\cap B_1)+Pr(A\cap B_2)+\cdots+Pr(A\cap B_n)$
Markov chains: $p_{i,j}$ the prob of moving from state i to state j at the next time step. The matrix $P=[p_{i,j}]$ is called the transition matrix. For each row of **P**, summing elements in a row gives 1. If X_n is a prob vector then $X_n=X_{n-1}P$ or $X_n=X_0P^n$. Factorial n denoted by $n!=1.2.3\ldots n$. The binomial coefficient $\binom{n}{k}=\frac{n!}{k!(n-k)}$ is the number of ways you can select k items out of n distinct items. Prob of k sucesses from n trials each with prob of success p is $P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$.

Summary statistics: Given a population of N values and sample from the population of $n<N$ values $x_1,\ldots,x_n$, sample mean: $\bar{x}=\frac{1}{n}\sum_{i=1}^n x_i$, sample variance: $s_x^2=\frac{1}{n-1}\sum_{i=1}^n(x_i-\bar{x})^2$, population mean: $\mu=\frac{1}{N}\sum_{i=1}^N x_i$, population variance: $\sigma^2=\frac{1}{N}\sum_{i=1}^N(x_i-\mu)^2$. Population standard deviation $\sigma=\sqrt{\sigma^2}$.

A discrete random variable X has expected value $E(X)=\sum x_iPr(x_i)$ and variance $var(X)=\sigma^2=\sum(x-\mu)^2Pr(x_i)$. Mode- most frequent does not have to be unique. Median given data ordered from smallest to largest, the median is the value that has half of the data below and half the data above it. If this point falls between two data points, the median is the average of the two values. **Normal distribution:** with mean μ and variance σ^2 is represented by $N(\mu,\sigma^2)$. "Bell shaped" curve that is symmetric about the mean (mean, median and mode are all the same). within $\pm$ 1/2/3 standard deviations of the mean there are 68%/ 95%/ 99.7% of the values. Total area under the curve is 1 and area between x_1 and x_2 ($x_1\leq x_2$) is $Pr(x_1\leq X\leq x_2)$. The standard normal, $N(0,1)$, has $\mu=0$ and $\sigma=1$. Given a value x from $N(\mu,\sigma^2)$, $P(X\leq x)=P(Z\leq z)$ where $z=\frac{x-\mu}{\sigma}$ is a value from $N(0,1)$ and $P(Z\leq z)$ is obtainable from a table or calculator. Binomial distribution $B(n,p)$ is approximately normal if p is not close to 0 or 1 and n large enough, with mean $\mu=np$ and standard deviation $\sigma=\sqrt{np(1-p)}$.

Two-Variable Calculus $z=f(x,y)$
Contour curve- the intersection of horizontal plane $z=k$ and $z=f(x,y)$. Level curve- projection of a contour curve onto the xy-plane. Partial derivative of f with respect to (w.r.t.) x, $f_x(x,y)$: treat y as constant and differentiate w.r.t. x. $f_x(x,y)=\lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}=\frac{\partial z}{\partial x}=\frac{\partial f}{\partial x}$. $f_x(x,y)$ is the slope of f at (x,y) in the direction of x. The partial derivative of f w.r.t. y, $f_y(x,y)$, is defined similarly. **Second order partial derivatives:** $f_{xx}(x,y)=\frac{\partial^2 f}{\partial x^2}$, $f_{yy}(x,y)=\frac{\partial^2 f}{\partial y^2}$. In general, $f_{xy}=\frac{\partial^2 f}{\partial x\partial y}=f_{yx}=\frac{\partial^2 f}{\partial y\partial x}$ this can be used to check calculations.
Critical points (a,b) are where both $f_x(a,b)=0$ and $f_y(a,b)=0$. Second derivative test, if (a,b) is a a critical point $\Delta(a,b)=f_{xx}(a,b)f_{yy}(a,b)-f_{xy}^2(a,b)=\left|\begin{array}{l} f_{xx}(a,b)f_{xy}(a,b)\\ f_{xy}(a,b)f_{yy}(a,b)\end{array}\right|_{(a,b)}$
f has a relative min at (a,b) if $\Delta(a,b)>0$ and $f_{xx}(a,b)>0$ relative max at (a,b) if $\Delta(a,b)>0$ and $f_{xx}(a,b)<0$ and Saddle pt. if $\Delta(a,b)<0$, test inconclusive if $\Delta(a,b)=0$. **chain rule for multivariable** fns $\frac{df}{dt}=\frac{\partial f}{\partial x}\frac{dx}{dt}+\frac{\partial f}{\partial y}\frac{dy}{dt}$
Lagrange Multipliers for constrained optimisation. Optimise $f(x,y)$ with constraint $g(x,y)=0$ form Lagrangian $L(x,y,\lambda)=f(x,y)+\lambda g(x,y)$ then find (a,b) such that $\frac{\partial L}{\partial x}=0$, $\frac{\partial L}{\partial y}=0$ and $\frac{\partial L}{\partial \lambda}=0$ this gives all possible points for maxima and minima.
Calculus
Average fn value between a and b is $\frac{f(b)-f(a)}{b-a}$ instantaneous rate of change is called the derivative. The derivative from first principles is $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. Marginal revenue is the derivative of revenue w.r.t q, similarly cost and profit. The derivative is also the slope of the tangent at a point. Second derivative, the result of differentiating a fn twice. Implicit dif-

ferentiation: treat y as a fn of x and use chain rule to differentiate leaving the derivative of y as $\frac{dy}{dx}$. f is increasing if $x_1 < x_2$ then $f(x_1) < f(x_2)$ or decreasing then $f(x_1) > f(x_2)$. If $f'(x) > 0$ then f is increasing, $f'(x) < 0$ then f is decreasing. Critical point $x = c$ occurs when $f'(c) = 0$ or $f'(c)$ is undefined. A critical point is a local max if $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ for $x > c$, conversley for minimum. Second derivative test, min if $f''(c) > 0$ and max if $f''(c) < 0$. when $f''(x) = 0$ test is inconclusive.
Pt. of inflection when $f''(x) = 0$. **Concavity**: if $f''(x) > 0$ f is concave up, or $f''(x) < 0$, concave down. If $(c, f(c))$ is a point of inflextion and f goes from concave up to concave down then c is the point of diminishing returns. $f(d)$ is the absolute max if $f(d) \geq f(x)$ $\forall x$, similarly for absolute min.
Profit = revenue - cost. Average cost = total cost divide quantity $c(x)/x$
Upper and Lower Sum, partition the interval (a, b) into n equal parts with interval width $\Delta x = \frac{b-a}{n}$ so the ith interval is $[x_{i-1}, x_i]$ where $x_i = a + i\Delta x$ M_i maximum on the ith interval and m_i the minimum on the ith interval. So $U_n = \Delta x \sum M_i$, $L_n = \Delta x \sum m_i$ if A is the area under the graph between a and b then $L_n \leq A \leq U_n$. An Estimate for A is $\frac{U_n+L_n}{2}$, with error in estimate $= \frac{U_n-L_n}{2}$. If f is increasing on (a, b) then $M_i = f(x_i)$ and $m_i = f(x_{i-1})$. **Definite integral** $\int_a^b f(x).dx = A$ the area under f between a and b. If $f(x) < 0$ then $\int_a^b f(x).dx < 0$ Area between fns is $\int (f(x) - g(x)).dx$ where $f(x) > g(x)$. **Fundamental theorem of Calculus** if $F'(x) = f(x)$ then $\int_a^b f(x).dx = F(b) - F(a)$ Average Fn value between (a, b) is given by $\frac{1}{b-a} \int_a^b f(x).dx$
p^*, q^* denote equilibirum price and qantity. **Consumer surplus**- the total amount saved by consumers buying at equilibrium price $= \int_0^{q^*} (D(q) - p^*).dq$.
Producer surplus, the benifit the producer gets by selling at equilibrium prices $= \int_0^{q^*} (p^* - S(q)).dq$
Present and future value, P present value, S future value, t years interest rate r compounded continuously so $P = Se^{-rt}$ or $S = Pe^{rt}$. Generalising if $f(t)$ represents a continous income stream in dollars for annum of k years, Present value of $f(t)$ is $P = \int_0^k f(t)e^{-rt}.dt$ and future value $S = e^{kr} \int_0^k f(t)e^{-rt}.dt$
Differential Equations (DE) an equation in which the Derivative of an unknown function occurs. Ordinary DE is a DE in which the unknown Fn is of 1 variable, and the order of a DE is the highest derivative that occurs. so if the DE is $f'(x) = kf(x)$ then $f(x) = Ae^{kx}$. Seperable DE $\frac{dy}{dx} = f(x)g(y)$ then find the solution by $\int (1/g(y)).dy = \int f(x).dx$. Matlab int command: $\int_a^b f(x).dx$ `syms x; int(f(x),x,a,b)` or for indefinite integral just `syms x; int(f(x),x)`.
Riemann sums, used to approximate integrals like upper and lower sums. Some examples are Left hand end pt.$LE_n = \Delta x \sum f(x_{i-1})$, Right hand end pt. $RE_n = \Delta x \sum f(x_i)$ and mid-point sum $M_n = \Delta x \sum f((x_i + x_{i-1})/2)$. Trapezoidal rule $T_n = (LE_n + RE_n)/2$. Error in approximation for mid-point and trapezoidal is roughly $1/n^2$

index laws	log laws
$a^n = a.a \ldots a$ n times	$\log(AB) = \log(A) + \log(B)$
$a^n a^m = a^{n+m}$	$\log(A/B) = \log(A) - \log(B)$
$a^n/a^m = a^{n-m}$	$\log A^n = n \log A$
$(a/b)^n = a^n/b^n$	$log_a b = \frac{\ln b}{\ln a} = \frac{\log_c b}{\log_c a}$
$a^{-n} = 1/a^n$ $\quad \sqrt[n]{a} = a^{1/n}$	$\log 1 = 0$

$\sum a_i + b_i = \sum a_i + \sum b_i$	$\sum_{i=1}^n 1 = n$
$\sum (ka_i) = k(\sum a_i)$	$\sum_{i=1}^n i = n(n+1)/2$
$\sum_{i=1}^k b_i + \sum_{i=k+1}^n b_i = \sum_{i=1}^n b_i$	$\sum_{i=1}^n i^2 = n(n+1)(2n+1)/6$

integral of f	f	df/dx
cx	c	0
$x^{n+1}/(n+1)$	x^n for $n \neq -1$	nx^{n-1}
$1/x$	$\ln\|x\|$	$-1/x^2$
e^x	e^x	e^x
$a^x/\ln a$	a^x	$a^x \ln a$
$k \int f(x)$	$f(x)$	$kf'(x)$
$\int f(x) + \int g(x)$	$f(x) + g(x)$	$f'(x) + g'(x)$

Quadratic law: if $ax^2 + bx + c = 0$ then $x = (-b \pm \sqrt{b^2 - 4ac})/2a$
product rule: $d/dx(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$
quotient rule: $d/dx(f(x)/g(x)) = (f'(x)g(x) - f(x)g'(x))/(g(x)^2)$
integration by parts: $\int uv'.dx = uv - \int u'v.dx$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad det(A) = ad - bc \qquad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$